Heim’s Mass Formula
(1982)

Original Text by Burkhard Heim
for the Programming of his Mass Formula

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On the Description of Elementary Particles

(Selected Results)

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A) <u>Invariants of Possible Basic Patterns (Multiplets)</u>

Symbols:

k	Configuration number, k = 0 : no ponderable particle (no rest mass). For ponderable particles only k = 1 and k = 2 possible, <u>not</u> k > 2. k is a metrical index number.
ε	so-called "time-helicity". Refering to the R_4 $\varepsilon = +1$ or $\varepsilon = -1$ decides whether it concerns an R_4 - structure or the mirror-symmetrical anti-structure ($\varepsilon = -1$).
G	the number of quasi-corpuscular internal sub-constituents of structural kind.
b_i	symbol for these $1 \le i \le G$ internal sub-constituents of an elementary particle.
B	baryonnumber
P	double isospin $P = 2s$.
$\underline{P}_{1,2}$	locations in P-interval, where multiplets appear multiplied (doubled).
I	number of components x of an isospin-multiplet, i.e. $1 \le x \le I$.
Q	double space-spin $Q = 2J$.
$\underline{Q}$	value of Q at $\underline{P}_{1,2}$.
$\kappa(\lambda)$	"doublet-number", which distinguishes between several doublets by $\kappa(\lambda) = 0$ or $\kappa(\lambda) = 1$.
Λ	Upper limit of κ-interval $1 \le \lambda \le \Lambda$.
C	structure-distributor, identical with sign of charge of the strangeness quantum number.
q_x	electrical charge quantum number with sign of the component x of the isospin-multiplet.
q	amount of charge quantum number $q = \lvert q_x \rvert$.

<u>Uniforme Description of Quantum Numbers by k und ε</u>

$$
\left.
\begin{aligned}
G &= k + 1 \\
B &= k - 1 \\
\underline{P}_1 &= 2 - k \\
\underline{P}_2 &= 2k - 1 \\
I &= P + 1 , \qquad 0 \le P \le G \\
Q(P) &= k - 1 \\
\underline{Q}(\underline{P}) &= 2k - 1 \\
\kappa(\lambda) &= (1 - \delta_{1\lambda}) \, \delta_{1P} , \qquad 1 \le \lambda \le \Lambda = 4 - k \\
C &= 2(P\varepsilon_P + Q\varepsilon_Q)(k - 1 + \kappa)/(1 + \kappa) \\
\varepsilon_{P,Q} &= \varepsilon \cos \alpha_{P,Q}
\end{aligned}
\right\} (I)
$$

$$\alpha_P = \pi Q(\kappa + \binom{P}{2})$$
$$\alpha_Q = \pi Q[Q(k-1) + \binom{P}{2}] \qquad \} \text{(II)}$$
$$2q_x = (P - 2x)[1 - \kappa Q(2 - k)] + \varepsilon[k - 1 - (1 + \kappa)Q(2 - k)] + C\,, \quad 0 \le x \le P\,, \quad q = |q_x|$$

Possible configurations $k = 1$, $k = 2$ with $\varepsilon = \pm 1$

Possible Multiplets of Basic States

Multiplet x_ν of serial number ν for $\varepsilon = +1$ and anti-multiplet $\bar{x}_n$ with $\varepsilon = -1$.

General Representation: $\bar{x}_n\,(\varepsilon B,\varepsilon P,\varepsilon Q,\varepsilon\kappa)_\varepsilon C(q_0,\ldots,q_P)$

Mesons: $k = 1$, $G = 2$ (quark?), $B = 0$, $0 \le P \le 2$, i.e from singlet $I = 1$ to triplet $I = 3$.
$Q = 0$, $\underline{Q} = 1$, $\Lambda(k{=}1) = 3$, $\kappa(1) = 0$, $\kappa(2) = \kappa(3) = 1$
Baryons: $k = 2$, $G = 3$ (quark?), $B = 1$, $0 \le P \le 3$ from singulett $I = 1$ to quartet $I = 4$,
$Q = 1$, $\underline{P}_1 = 0$, $\underline{P}_2 = 3$, $\underline{Q} = 3$, $\Lambda(k{=}2) = 2$, $\kappa(1) = 0$, $\kappa(2) = 1$

Possible multipletts for $\varepsilon = +1$:

$k = 1$: x_1 (0000)0(0) ≡ (η)
x_2 (0110)0(0,-1) ≡ (e_0,e^-), (is the existence of e_0 possible ?)
x_3 (0111)0(-1,-1) ≡ x_3 (0111)0(-1) ≡ (μ^-) pseudo-singlet }(III)
x_4 (0101)+1(+1,0) ≡ (K^+, K^0)
x_5 (0200)0(+1,0,-1) ≡ x_5 (0200)0(±1,0) ≡ ($\pi^\pm$, π^0) anti-triplet to itself

$k = 2$: x_6 (1010)-1(0) ≡ (Λ)
x_7 (1030)-3(-1) ≡ (Ω^-)
x_8 (1110)0(+1,0) ≡ (p,n)
x_9 (1111)-2(0,-1) ≡ (Ξ^0,Ξ^-) }(IV)
x_{10} (1210)-1(+1,0,-1) ≡ (Σ^+,Σ^0,Σ^-)
x_{11} (1310)-2(+1,0,-1,-2) ≡ (o^+,o^0,o^-,o^{--}), (existence possible ?)
x_{12} (1330)0(+2,+1,0,-1) ≡ (Δ^{++}, Δ^+, Δ^0, Δ^-), (thinkable as a basic state ?)

Abbreviations:

$$\eta = \pi/(\pi^4 + 4)^{1/4}$$
$$\eta_{kq} = \pi/[\pi^4 + (4+k)q^4]^{1/4}$$
$$\vartheta = 5\,\eta + 2\sqrt{\eta} + 1 \qquad \}\text{(V)}$$
$$A_1 = \sqrt{\eta_{11}}\,(1 - \sqrt{\eta_{11}})/\,(1 + \sqrt{\eta_{11}})$$
$$A_2 = \sqrt{\eta_{12}}\,(1 - \sqrt{\eta_{12}})/\,(1 + \sqrt{\eta_{12}})$$

Planck's constant: $\hbar = h/2\pi$, light-velocity: $c = (\varepsilon_0\mu_0)^{-1/2}$, wave-resistance of empty space R_3 (electro-magnetic): $R_- = c\mu_0$, with ε_0 and μ_0 constants of influence and induction.
Elektrical elementary charge: $e_\pm = 3C_\pm$ with

$$C_\pm = \pm\sqrt{2J\hbar / R_-}\ /(4\pi)^2 \qquad \text{(possibly electr. quark-charge ?)}$$

Finestructure-constant: $\alpha\sqrt{(1-\alpha^2)} = 9\vartheta\,(1 - A_1A_2) / (2\pi)^5$, $\alpha > 0$.

Solution: $\alpha_{(+)}$ (positive branch) and $\alpha_{(-)}$ (negative branch).

Numerical: $\alpha_{(+)}^{-1} = 137{,}03596147$
$\alpha_{(-)}^{-1} = 1{,}00001363$

[A better formula, 1992, yields $\alpha_{(+)} = 1/137{,}0360085$ and $\alpha_{(-)\_} = 1/1{,}000026627$]

What is the meaning of that strong coupling $\alpha_{(-)}$?
Abbreviation: $\alpha_{(+)} = \alpha$, $\alpha_{(-)} = ß \approx 137\,\alpha$.

B) Mass-Spectrum of Basic Patterns and its Resonances

Used constants of nature and pure numbers:

Planck's constant: $\hbar = h/2\pi = 1{,}0545887 \times 10^{-34}$ J s,
light-velocity: $c = 2{,}99792458 \times 10^8$ m s^{-1},
Newton's constant of gravitation: $\gamma = 6{,}6732 \times 10^{11}$ N m^2 kg^{-2}
constant of influence $\varepsilon_0 = 8{,}8542 \times 10^{-12}$ A sV^{-1} m^{-1},
constant of induction $\mu_0 = 1{,}2566 \times 10^{-6}$ A^{-1} s V m^{-1},
vacuum wave-resistance $R_- = (\mu_0/\varepsilon_0)^{1/2} = 376{,}73037659$ V A^{-1}

derived constants of nature (mass-element):

$$m = \sqrt[4]{p}\,\sqrt[3]{3pg\hbar s_0}\,\sqrt{\hbar / 3cg}\,s_0^{-1} \ , \qquad s_0 = 1\,[\mathrm{m}]\ \text{(gauge factor)} \qquad \text{(VI)}$$

Basis of natural logarithms: e = 2,71828183
number $\pi = 3{,}1415926535$
geometrical constant: $\xi = 1{,}61803399$
[Limes of the "creation-selector"] $\lim_{n\to\infty} a_n : a_{n-1} = \xi$ by the series $a_n = a_{n-1} + a_{n-2}$.
(till the 8th decimal place, represented by $\xi = (1 + \sqrt{5})/2$).

Auxiliary functions:

$$\eta = \pi/(\pi^4 + 4)^{1/4} \qquad \text{(VII)}$$

$$t = 1 - 2/3\,\xi\,\eta^2\,(1 - \sqrt{\eta})$$
$$\alpha_+ = t\,(\eta^2\,\eta^{1/3})^{-1} - 1\} \qquad \text{(VIII)}$$
$$\alpha_- = t\,(\eta\eta^{1/3})^{-1} - 1$$

Quantum numbers by (A):

$$\eta_{qk} = \pi/[\pi^4 + (4+k)q^4]^{1/4}$$

$$N_1 = \alpha_1$$
$$N_2 = (2/3)\,\alpha_2\,,$$
$$N_3 = 2\,\alpha_3\,,$$

with

$$\left.\begin{aligned} \alpha_1 &= \tfrac{1}{2}\,(1 + \sqrt{\eta_{qk}}\,)\,, \\ \alpha_2 &= 1/\,\eta_{qk}\,, \\ \alpha_3 &= e^{(k-1)}/k - q\,\{\alpha/3\,[(1 + \sqrt{\eta_{qk}}\,)(\xi/\eta_{qk}{}^2)]^{(2k+1)}\,\eta_{qk}{}^3 + \\ &\quad + [\eta(1,1)/\,e\,\eta_{qk}]\,(2\sqrt{\xi\eta_{qk}})^k\,[(1 - \sqrt{\eta_{qk}})/(1 + \sqrt{\eta_{qk}})]^2\,\} \end{aligned}\right\}\text{(IX)}$$

Invariants of metrical steps-structure (abbreviation $s = k^2 + 1$):

$$\left.\begin{aligned} Q_1 &= 3 \cdot 2^{s-2}\,, \\ Q_2 &= 2^s - 1\,, \\ Q_3 &= 2^s + 2(-1)^k\,, \\ Q_4 &= 2^{s-1} - 1\,. \end{aligned}\right\}\text{(X)}$$

Fourfold R_3-construct $1 \le j \le 4$. Q_j = const. with respect to time t. Parameter of occupation $n_j = n_j(t)$ caused radioactive decay. Mass elements of occupations of the configurations zones j are $\mu\alpha_+$.

Further auxiliary functions of zones occupations:

$$\left.\begin{aligned} K &= n_1{}^2\,(1+n_1)^2N_1 + n_2\,(2n_2{}^2+3n_2+1)N_2 + n_3\,(1+n_3)N_3 + 4n_4\,, \\ \underline{G} &= Q_1{}^2(1+Q_1)^2N_1 + Q_2(2Q_2{}^2+3Q_2+1)N_2 + Q_3(1+Q_3)N_3 + 4Q_4\,, \\ H &= 2n_1Q_1[1+3(n_1+Q_1+n_1Q_1) + 2(n_1{}^2+Q_1{}^2)]N_1 + 6n_2Q_2(1+n_2+Q_2)N_2 + 2n_3Q_3N_3 \end{aligned}\right\}\text{(XI)}$$

$$\begin{aligned} \Phi = {} & 3\,P/(\pi\sqrt{\eta_{qk}})\,(1 - \alpha_-/\alpha_+)(P+Q)(-1)^{P+Q}[1-\alpha/3+\pi/2\,(k-1)\,3^{1-q/2}\,] \\ & *\{1+2\,k\,\kappa/(3\,\eta^2)\,\xi[1 + \xi^2(P-Q)(\pi^2-q)]\}\,[1 +(\,4\,\xi\left(\tfrac{P}{2}\right)/k)(\xi\,/6)^q]^{-1} \\ & *[\,2\sqrt{\eta_{11}}\sqrt{\eta_{qk}} + q\eta^2\,(k - 1)]\,(1+4\pi\alpha/\eta\sqrt{\eta})(1+Q(1-\kappa)(2-k)n_1/Q_1] \\ & + 4\,(1 - \alpha_-/\alpha_+)\alpha(P+Q)/\xi^2 \;+ 4\,q\alpha_-/\alpha_+ \end{aligned}$$

<u>Uniform Mass spectrum:</u>

$$M = \mu\alpha_+\,(K + \underline{G} + H + \Phi) \qquad \text{(XII)}$$

Not each quadruple n_j yields a real mass! To the selection rule: in the fourfold R_3-construct $1 \le j \le 4$ configurations zones n(j=1), m(j=2), p(j=3), σ(j=4). Increase of occupation with metrical structure elements:

<u>central zone</u> n cubic,
<u>internal zone</u> m quadratic,
<u>meso-zone</u> p linear (continuation to the empty space R_3),
<u>external zone</u> σ selective.

Principle of increase of the configurations zones:

$$n_4+Q_4 \le (n_3+Q_3)\alpha_3 \le (n_2+Q_2)^2\,\alpha_2 \le (n_1+Q_1)^3\alpha_3 \qquad \text{(XIII)}$$

<u>Selection rule for the Occupation of Configuration Zones</u>

$$(n_1+Q_1)^3\alpha_1 + (n_2+Q_2)^2\,\alpha_2 + (n_3+Q_3)\alpha_3 + \exp[1-2k(n_4+Q_4)/3Q_4] + iF(\Gamma) = \qquad \text{(XIV)}$$
$$= W_{\nu x}\{1 + [1-Q(2-k)(1-\kappa)][a_{\nu x}N/(N+2) + b_{\nu x}\sqrt{N(N-2)}\ ]\}.$$

$$W_{\nu x} = g(qk)\, w_{\nu x}\ ,$$

Basis rise: $g(qk) = Q_1^3\alpha_1 + Q_2^2\,\alpha_2 + Q_3\alpha_3 + \exp[(1-2k)/3]$ for $n_j = 0$. (XV)

Structure power of the discussed state $w_{\nu x} = (kPQ\kappa)_\varepsilon C(q_x)$ as component x of multiplets ν is:

$$w_{\nu x} = \Big\{(1-Q)[A_{11}-P(A_{12}+A_{13}q\kappa/\eta_{qk}) - \binom{P}{2}(A_{14}-A_{15}q/\eta_{qk})] + \kappa Q\eta_{qk}A_{16}\Big\}^{2-k} +$$
$$+ \Big\{(q-1)A_{21} + (1-P)A_{22} + \binom{P}{2}[A_{23}-q_x\eta_{qk}(1+A_{24}(+q_x))^{-1}A_{25}] + \qquad \Big\}\text{(XVI)}$$
$$+ \kappa(A_{26}+q\eta_{qk}^2A_{31}) + \binom{Q}{3}\eta_{qk}A_{32} + \binom{P}{3}[A_{33}q^3(q_x - (-1)^q)/(3-q) +$$
$$+ \frac{e(P-Q)h^{(q+1)q/4}}{8-A_{66}^{q(q-1)}}\,(1 - q(2-q)A_{34}{}^{1-q}{}_xA_{35}/\eta_{qk})\,\eta_{qk}/\eta^2 - A_{36}]\Big\}^{k-1}.$$

$$w(1) = (1-Q)[A_{11} - P(A_{12}+ A_{13}q\kappa/\eta_{qk}) - \binom{P}{2}(A_{14} - A_{15}q/\eta_{qk})] + \kappa Q\,\eta_{qk}A_{16} \qquad \text{(XVII)}$$

and

$$w(2) = (q-1)A_{21} + (1-P)A_{22} + \binom{P}{2}[A_{23} - A_{25}q_x\eta_{qk}(1 + A_{24}(1+q_x))^{-1}] +$$
$$+ \kappa(A_{26} + q\eta_{qk}^2A_{31}) + \binom{Q}{3}\eta_{qk}A_{32} + \binom{P}{3}\{A_{33}q^3[q_x - (-1)^q)/(3-q)] + \qquad \text{(XVIII)}$$
$$+ \frac{e(P-Q)h^{(q+1)q/4}}{8-A_{66}^{q(q-1)}}\,[1 - q(2-q)A_{34}{}^{1-q}{}_xA_{35}/\eta_{qk}]\,\eta_{qk}/\eta^2 - A_{36}\}$$

in $$w_{\nu x} = [w(1)]^{2-k} + [w(2)]^{k-1} \qquad \text{(XIX)}$$

can become $w(2) = 0$ for single sets of quantum numbers at $k = 1$ or $w(1) = 0$ at $k = 2$, which leads to terms 0^0, which but must have always have the value 1 as parts of structure power. Therefore it is recommended for programming to complete w(1) and w(2) by the numerical non-relevant summands k-1 and 2-k . Since always $w(1) \neq -1$ and $w(2) \neq -1$ remain, but only k=1 or k=2 is possible, the actually terms in the expression

$$w_{\nu x}(k) = [k-1+w(1)]^{2-k} + [2-k+w(2)]^{k-1}$$

do no more appear. By this correction it is evident that for mesonical structures $w_{\nu x}(k=1) = 1 + w(1)$ and for barionical structures $w_{\nu x}(k=2) = 1 + w(2)$ holds.

As a basis of resonance holds $a_{vx} = A_{41} (1 + a_n a_q)/k$ (XX)

with $a_n = PA_{42} [1 - \kappa A_{43} (1 + A_{44} (-\alpha)^{2-k} A_{45}^{k-1})*$
$*(1 - \kappa Q A_{46}(2-k)) - A_{51}(k-1)(1-\kappa)]$ (XXI)

and $a_q = 1 - qA_{52}(1 - 2A_{53}^{k})[1 + q_x(3-q_x)(k-1)(1-\kappa)/6]$ (XXII)

Resonance grid is

$$b_{vx} = \{A_{54}A_{55}^{k-1} [1 - PA_{56}(1-\kappa A_{61}A_{62}^{1-k})(1 + qA_{63}(1 + \kappa A_{64}))] * \quad \text{(XXIII)}$$
$$* (1-k^{-1} (A_{65}(q+k-1))^{2-k} \binom{P}{2} (1 - \binom{P}{3}))\}/[k^P(1+P+Q+\kappa\eta^{2-q})] .$$

The coefficients A_{rs} can be seen as elements of the quadratic coefficient matrix $\hat{A} = (A_{rs})_6$ with $A_{rs} \neq A_{sr}$ and $\mathrm{Im} A_{rs} = 0$.

Proposal for the determination of matrix elements (reduction to π, e and ξ):

$A_{11} = (\xi^2 \pi e)^2 (1 - 4 \pi \alpha^2) / 2 \eta^2$,
$A_{12} = 2 \pi \xi^2 (\vartheta/24 - e \pi \eta \alpha^2 / 9)$
$A_{13} = 3 (4 + \eta \alpha)[1 - (\eta^2/5)((1 - \sqrt{\eta})^2 /(1 + \sqrt{\eta})^2]$
$A_{14} = [1 + 3 \eta (2 \eta \alpha - e^2\xi(1 - \sqrt{\eta})^2/(1 + \sqrt{\eta})^2)/4\xi]/ \alpha$

$A_{15} = e^2(1 - 2e\alpha^2/\eta)/3$
$A_{16} = (\pi e)^2[1 + \alpha(1+6\alpha/\pi)/5\eta]$
$A_{21} = 2(e\alpha/2\eta)^2(1 - \alpha/2\xi^2)$
$A_{22} = \xi[1 - \xi(\alpha\xi/\eta^2)^2]/12$

$A_{23} = (\eta^2 + 6\xi\alpha^2)/e$
$A_{24} = 2\xi^2/3\eta$
$A_{25} = \xi(\pi e)^2(1 - ß^2)$
$A_{26} = 2\{1 - [\pi(e\xi\alpha)^2\sqrt{\eta}]/2\}/e\xi^2$

$A_{31} = (\pi e\alpha)^2[1 - (\pi e)^2(1 - ß^2)]$
$A_{32} = \xi^2[1 + (2e\alpha/\eta)^2]/6$
$A_{33} = (\pi e\xi\alpha)^2[1 - 2\pi(e\xi)^2(1 - ß^2)]$
$A_{34} = \eta \sqrt{2ph}$
(XXIV)
$A_{35} = 3\alpha/e\xi^2$
$A_{36} = [1 - \pi e(\xi e)^2(1 - ß^2)]^{-1}$
$A_{41} = \{\xi[2 + (\xi\alpha)^2] - 2ß\}/(2ß - \alpha)$
$A_{42} = [\pi\xi^2\eta(ß - 3\alpha)]/2$

$A_{43} = \xi/2$
$A_{44} = 2(\eta/\xi)^2$
$A_{45} = (3ß - \alpha)/6\xi$
$A_{46} = \pi e/\xi\eta - e\eta^2\alpha/2$

$A_{51} = (2\alpha + 1)^2$
$A_{52} = 6\alpha/\eta^2$
$A_{53} = (\xi/\eta)^3$
$A_{54} = \alpha(ß - \alpha)\sqrt{(3/2)}$

$A_{55} = \xi^2$
$A_{56} = (\xi/\eta)^4$
$A_{61} = \pi\xi(2ß - \alpha)/12ß$
$A_{62} = \pi^2(ß - 2\alpha)/12$

$A_{63} = (\sqrt{\eta})/9$
$A_{64} = \pi/3\eta$
$A_{65} = \pi/3\xi$
$A_{66} = \xi\eta$

The order of resonance $N \geq 0$ (positive integer) selects the admitted quadruple n_j with $1 \leq j \leq 4$. With

$$f(N) = [1 - Q(2 - k)(1 - \kappa)][a_{\nu x} N/(N+2) + b_{\nu x} \sqrt{N(N-2)}\,] \qquad \text{(XXV)}$$

follows that the unknown function $F(\Gamma)$ remains 0 for all $N \neq 1$ (right side is real). In the case of $N = 0$ is $f = 0$, so that

$$(n_1 + Q_1)^3\alpha_1 + (n_2 + Q_2)^2 \alpha_2 + (n_3 + Q_3)\alpha_3 + \exp[(1\text{-}2k)(n_4+Q_4)/3Q_4] = W_{\nu x} \quad \text{(XXVI)}$$

describes the n_j of the state $x_{\nu x}$ and hence the mass $M_0(\nu x)$ of the component x of the multiplet x_ν . The $N \geq 2$ assign $x_{\nu x}$ to a spectrum of occupation-parameter quadruples and with that, according to the mass-formula, resonance-masses $M_N(\nu x)$ (for each component $x_{\nu x}$ a spectrum of masses). In the case of $N = 1$ no spectral term. Here is not $f(N) \geq 0$, $f(1)$ is complex.

Real part: $(\underline{n}_1+Q_1)^3\alpha_1 + (\underline{n}_2+Q_2)^2 \alpha_2 + (\underline{n}_3+Q_3)\alpha_3 + \exp[(1\text{-}2k)(\underline{n}_4+Q_4)/3Q_4] =$
$= W_{\nu x}\{1+[1\text{-}Q(2\text{-}k)(1\text{-}\kappa)]a_{\nu x}/3\}$ (XXVII)

Imaginary part $F(\Gamma) = W_{\nu x}[1\text{-}Q(2\text{-}k)(1\text{-}\kappa)]b_{\nu x}$. (XXVIII)

The $\underline{n}_j$ and $F(\Gamma)$ are somehow related with N to the complete bandwidths Γ . Also there must be a connection $Q_N = Q(N)$ between doubled spin quantum-number Q and N . How could this connection be like?

If $N = 1$ is excluded, then $F = 0$, and the real relationship

$$(n_1 + Q_1)^3\alpha_1 + (n_2 + Q_2)^2 \alpha_2 + (n_3 + Q_3)\alpha_3 + \exp[(1\text{-}2k)(n_4+Q_4)/3Q_4] = W_{\nu x}(1+f) \quad \text{(XXIX)}$$

has to be discussed. Generally $f > 0$ for $N \geq 2$ and $f = 0$ for $N = 0$. But in the case of the multiplets x_2 $f = 0$ for all $N \geq 0$, since only here is $Q(2\text{-}k)(1\text{-}\kappa) = 1$. Electrons according to this image can not be stimulated !

For a numerical evaluation of $W_{\nu x}$, $a_{\nu x}$, $b_{\nu x}$ and $\Phi_{\nu x}$ (quantum number function in mass spectrum M) not Q_N = Q(N) , but use Q = Q(0) of x_ν . For the evaluation of n_j the principle of increase of the occupations of configuration zones is considered. First determine the right side $W_{\nu x}$ (1+f(N)) = W_1 numerically for an order of resonance N = 0 or N ≥ 2 . Determine according to the selection rule the maximal cubic number K_1^3 whose product with α_1 is contained in W_1 . Then insert $W_1 - \alpha_1 K_1^3 = W_2 \geq 0$ into

$$(n_2 + Q_2)^2 \alpha_2 + (n_3 + Q_3)\alpha_3 + \exp[(1-2k)(n_4+Q_4)/3Q_4] = W_2 \,. \qquad \text{(XXX)}$$

Now maximal quadratic number K_2^2 such, that $\alpha_2 K_2^2$ is still a factor of W_2 , i.e. $W_2 - \alpha_2 K_2^2 = W_3 \geq 0$. Accordingly in

$$(n_3 + Q_3)\alpha_3 + \exp[(1-2k)(n_4+Q_4)/3Q_4] = W_3 \qquad \text{(XXXI)}$$

Determine maximal number K_3 in the way $W_3 - \alpha_3 K_3 = W_4 \geq 0$.

Three possibilities for W_4 :
(a): $W_4 = 0$,
(b): $0 < W_4 \leq 1$,
(c): $W_4 > 1$.

General case (b): $\ln W_4 \leq 0$ and $K_4(2k-1) = -3Q_4 \ln W_4$.
In case of (c) it is $\ln W_4 > 0$ and K < 0 . This is impossible, since always $n_j+Q_j \geq 0$ has to be.
According to $n_4+Q_4 \leq (n_3+Q_3)\alpha_3$ of the principle of rise K_3 will be lowered by 1 and $\alpha_3 K_3$ is added to $K_4 < 0$, so that a new value $K_4 \geq 0$ will be generated., which requires $K_3 > 0$, since in that case $K_3 = 0$. This dilatation can not happen because of the quadratic rise of j = 2 , so that this order of resonance N does not exist for $x_{\nu x}$ (forbidden term).

In the case (a) $W_4 \to 0$ would have as a consequence the divergence $K_4 \to \infty$, but this is impossible according to $K_4 \leq \alpha_3 K_3$ (particularly there are no diverging self-potentials). For that reason will be calculated in case of (a) the maximal value $K_4 = \alpha_3 K_3$. From the computed K_j it follows $n_j = K_j - Q_j$. Beside $n_j \geq 0$ also $n_j < 0$ is possible , but it holds always $K_j \geq 0$, i.e. $n_j \geq -Q_j$. The quadruple n_j determined in that way will be inserted with $\Phi_{\nu x}$ in the spectrum of masses, which numerically yields $M_N(\nu x)$ as a spectral-term of mass-spectrum at $x_{\nu x}$.

Note: The K_j are always integers. But in the case of the evaluation of K_4 generally decimal figures will occur. In case of the decimal places ,99... $\overline{99}$ one has to use the identity ,99... $\overline{99}$ = 1 . But if the series of decimal places is different from this value, then one has not to round up. The decimal places are to cut off , since the K_j are the numbers of structure entities.

Limits of Resonance Spectra

General construction-principle of configuration-zones

$$n_4+Q_4 \leq (n_3+Q_3)\alpha_3 ,$$
$$\alpha_3 (n_3+Q_3)(1+n_3+Q_3) \leq 2\alpha_2(n_2+Q_2)^2 , \qquad \text{(XXXII)}$$
$$\alpha_2 (n_2+Q_2)[2(n_2+Q_2)^2 + 3(n_2+Q_2) + 1] \leq 6\alpha_1 (n_1+Q_1)^3 .$$

If by the increase of N between two zones equality is reached, then $n_j+Q_j \rightarrow 0$ in j, while j-1 will be raised by 1 to $n_{j-1} + Q_{j-1} + 1$. The stimulation takes place “from outside to the interior“. Always $n_j+Q_j \geq 0$ is an integer, since they are the numbers of structure entities. Empty-space-condition: $n_j = -Q_j$, but $(n_j)_{max} = L_j < \infty$ (no diverging self-energy potentials). Intervals $-Q_j \leq n_j \leq L_j < \infty$ cause $0 \leq N \leq L < \infty$ of resonance-order. With $M_0(\nu x) = M_0$ holds

$$4\mu\alpha_+\alpha_1 (L_1+Q_1)^3 = [2(P+1)]^{2-k}M_0G \qquad \text{(XXXIII)}$$

with G = k+1 and from that by the construction-principle

$$\alpha_2 (L_2+Q_2)[2(L_2+Q_2)^2 + 3(L_2+Q_2) + 1] \leq 6\alpha_1 (L_1+Q_1)^3 ,$$
$$\alpha_3 (L_3+Q_3)(1+L_3+Q_3) \leq 2\alpha_2(L_2+Q_2)^2 , \qquad \text{(XXXIV)}$$
$$L_4+Q_4 \leq (L_3+Q_3)\alpha_3 .$$

For L implicitly the resonance-order is

$$(L_1 + Q_1)^3\alpha_1 + (L_2 + Q_2)^2 \alpha_2 + (L_3 + Q_3) \alpha_3 + \exp[(1-2k)(L_4+Q_4)/3Q_4] =$$
$$=W_{\nu x} [1+f(L)] \qquad \text{(XXXV)}$$

Also in the evaluation of L_j and L do <u>not</u> round up, but cut off decimal digits! The L_j which are obtained by the construction-principle, yield the absolute maximal masses M_{max} , and the quadruples, which are obtained from the L, yield the real limit-terms $M_L < M_{max}$, which are to stimulate secondaryly with $(M_{max} - M_L)c^2$ and then reach M_{max} .

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