Heim's Mass Formula (1989)

According to a Report by Burkhard Heim

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Content

Introduction

After DESY physicists in 1982 had programmed and calculated the mass formula which was published in the book *Elementarstrukturen der Materie* (Heim 1984), the mentioned formula by B. Heim was extended and in 1989 a 57 pages report with a new formula and the results of the calculations were sent to the company MBB/DASA. Unfortunately this later code could no more be recovered today.

Parts of these formulae have now been programmed again by the research group „Heim Theory" (by Dr. A. Mueller). It was found that in the manuscript some brackets in very long equations were lost during the process of writing; this had to be corrected at best estimate. The code covers the masses of basic states only and no lifetimes.

Other than the program written in 1982, Heim's 1989 computation also includes the life times of the basic states, the neutrino masses, and the finestructure constant. Therefore, these equations shall be given here, as far as they deviate from those given in the manuscript in 1982.

The *structure distributor* C (i.e. strangeness) given in *eq.* (I) *of chapt. E* has to be divided by k. One of the angles by which the *time helicity* ε is defined must read

$$\alpha_Q = \pi Q \left[Q + \binom{P}{2}\right] \tag{B1}$$

The expression for the quantum number of charge other than in (II) now reads:

$$q_x = \tfrac{1}{2}\Big[(P - 2x + 2)\,[1 - \kappa Q(2 - k)] + \varepsilon[k - 1 - (1 + \kappa)Q(2 - k)] + C \Big] \tag{B2}$$

All other constants are defined by eq.(I).

1. Mass of Basic States and of the Excited States of Elementary Particles

The modified **mass formula** of elementary particles is built up - other that in eq.(XII) - by the following parts:

$$M = \mu\alpha_+ \left[(G + S + F + \Phi) + 4\, q\, \alpha_-\right] \tag{B3}$$

The parts G and S are the same as G and K in eq.(XII) (now using n, m, p instead of n_1, n_2, n_3); μ is the mass element as in eq.(VI). The constants $\alpha_\pm$ have the form:

$$\alpha_+ = \frac{\sqrt[6]{h}}{h^2}\left(1 - J\left[\frac{2(1-\sqrt{h})}{h(1+\sqrt{h})}\right]^2 \sqrt{2h}\right) - 1\,, \quad \alpha_- = (\alpha_+ + 1)\eta - 1 \tag{B4}$$

The calculated results for α_+ and α_- in (B4) are shown in a table VI/chapterG.

The abbreviations for F and Φ, which depend on the quantum numbers, read:

$$F = 2\, n\, Q_n \left[1 + 3(n + Q_n + n\, Q_n) + 2(n^2 + Q_n^2)\right] + \\ + 6\, m\, Q_m\, (1 + m + Q_m)N_2 + 2\, p\, Q_p\, N_3 + \varphi\,(p,\sigma) * \delta(N) \tag{B5}$$

$$\Phi = P(-1)^{P+Q} (P + Q) N_5 + Q(P + 1) N_6 \tag{B6}$$

$$\varphi = \varphi (p,\sigma) , \qquad \delta(0) = 1 \ (0 \text{ for } N \neq 0) \tag{B7}$$

with

$$\varphi = \frac{N_4 p^2}{1+p^2} \frac{s + Q_s}{\sqrt{1+s^2}} (\sqrt[4]{2} - 4BUW_{N=0}^{-1}) + P(P - 2)^2(1 + \kappa(1 - q)/2\alpha J)(\pi/e)^2\sqrt{\eta_{12}}(Q_m-Q_n) -$$
$$- (P + 1)\binom{Q}{3}/\alpha , \qquad \text{(comp. with eq. B49)}$$

$$U = 2^Z [P^2 + 3/2 (P - Q) + P(1 -q) + 4\kappa B (1 -Q)/(3 - 2q) +$$
$$+ (k - 1)\{P + 2Q - 4\pi(P - Q)(1 - q)/\sqrt[4]{2} \}] \eta_{qk}^{-2} \qquad \text{(comp. with eq. B50)}$$

and $$Z = k + P + Q + \kappa \qquad \text{(comp. with eq. B51)}$$

φ is a term of self-couplings, which depends on p and σ and essentially determines the life time of a basic state. φ appears only in the basic states; therefore the symbol δ(N) as a unit element is used. The functions Q_i from eq. (X) remain unchanged. For n_1, n_2, n_3, n_4 in eq. (B5) here n, m, p, σ will be written. The constants $\eta_{q,k}$, J and η (with $\eta_{10} = \eta$, and $J_{1,0} = J$), as well as the functions N_1 and N_2 read as in eq.(IX). The remaining N_i with i > 2 are:

$$\ln (N_3 k/2) = (k -1) [1 - \pi\frac{1-h_{q,k}}{1+\sqrt{h_{q,1}}}\{1-u\frac{h_{q,1}}{J_{q,1}}(1-a_- / a_+)(1-\sqrt{h})^2\}] -$$
$$- 2/(3\pi e) (1 - \sqrt{h})^2 (6 \pi^2 e^2/ J \ \frac{1+\sqrt{h_{q,1}}}{1-h} - 1) \tag{B8}$$

$$N_4 = (4/k) [1 + q(k - 1)] \tag{B9}$$

$$N_5 = A[1 + k(k - 1) 2^{k^2+3} N(k) A \left(\frac{1-\sqrt{h_{q,k}}}{1+\sqrt{h_{q,k}}}\right)^2] \tag{B10}$$

$$A = (8/\eta)(1 - \alpha_-/\alpha_+)(1 - 3\eta/4) \tag{B11}$$

$$N(k) = Q_n + Q_m + Q_p + Q_\sigma + k(-1)^k 2^{k^2-1} \tag{B12}$$

$$N_6 = 2k/(\pi e J) [\sqrt{k} (k^2 - 1) \frac{N(k)}{\sqrt{h_{1,k}}}\{q - (1 - q) \frac{N'(k)}{Q_n\sqrt{h_{1,k}}}\} +$$
$$+ (-1)^{k+1}] \eta(1 - a_- / a_+)\left(4\frac{1-\sqrt{h}}{1+\sqrt{h}}\right)^2 Q_\sigma \tag{B13}$$

$$N'(k) = Q_n + Q_m + Q_p + Q_\sigma - 2k -1 \tag{B14}$$

The calculated results for B8, B9, B10 and B13 can be found in a table VII/chapter G.

Let L be the upper barrier such that as soon as it is reached the filling of the zone x disappears and the foregoing zone filling of next higher order is raised by 1. With the symbols $L_{(x)}(x - 1)$ for this barrier and $M_0 = M(N=0)$ the limits of the fillings of structure zones corresponding to eq.(XXXIII) are given by:

$$- Q_n \leq n \leq L_{(n)} = \frac{\sqrt[3]{(P+1)M_0}}{2ma_+N_1} - Q_n \quad \text{(B15)}$$

since in the case of the central region there are no further fillings.
For the series of numbers m the limitation holds:

$$- Q_m \leq m \leq L_{(m)}(n) \quad \text{(B16)}$$

with $2(Q_m + L_{(m)}(n))^3 + 3(Q_m + L_{(m)}(n))^2 + Q_m + L_m(n) = 4 N_1(n + Q_n)^3/N_2$ (B17)

Correspondingly, we have

$$- Q_p \leq p \leq L_{(p)}(m) \quad \text{(B18)}$$

with $2 L_{(p)}(m) = \sqrt{24\frac{N_2}{N_3}(m+Q_m)^2+1} - 2Qp - 1$ (B19)

and $- Q_\sigma \leq \sigma \leq L_{(\sigma)}(p)$ (B20)

with $2 L_{(\sigma)}(p) = N_3(p + Q_p) - 2 Q_\sigma$ (B21)

The calculated results for B15 can be found in a table IX/chapter G.

The selection rule which expresses the n, m, p, σ by the quantum numbers k, P, Q, κ, q and N, is described by eq.(XXIX).

In that f(N) is the *excitation function* for N > 0. For the factor $W_{\nu x} \equiv W_{N=0}$, which is independent of the exciting state, holds:

$$W_{N=0} = A e^x (1 - \eta)^L + (P - Q)(1 - \tbinom{P}{2})(1 - \tbinom{Q}{3})(1 - \sqrt{h})^2 \sqrt{2} \quad \text{(B22)}$$

with

$$A = 8 g H[2 - k + 8H (k - 1)]^{-1} \quad \text{(B23)}$$
$$H = Q_n + Q_m + Q_p + Q_\sigma \quad \text{(B24)}$$
$$g = Q_n^2 + Q_m^2 + (Q_p^2/k) \; e^{k-1} + \exp[(1- 2k)/3] - H(k - 1) \quad \text{(B25)}$$
$$L = (1 - \kappa) Q (2 - k) \quad \text{(B26)}$$
$$x = [1 - Q - \tbinom{P}{2}](2 - k) + 1/4B [a_1 + k^3/(4H)(a_2 + a_3/(4B))] \quad \text{(B27)}$$
$$B = 3 H [k^2 (2k - 1)]^{-1} \quad \text{(B28)}$$

The calculated results for B23, B24, and B28 can be found in a table VI/chapter G.

For the three parameters a_1, a_2, and a_3 the following combinatorical relations hold:

$$a_1 = 1 + B + k(Q^2 + 1)\binom{Q}{3} - \kappa[(B - 1)(2 - k) - 3\{H - 2(1 + q)\}(P - Q) + 1] - (1 - \kappa)\Big[\Big(3(2 - q)\binom{P}{2} - Q\{3(P + Q) + q\}\Big)(2 - k) + [k(P + 1)\binom{P}{2} + \{1 + B/k\,(k + P - Q)\}(1 - \binom{P}{2})(1 - \binom{Q}{3}) - q(1 - q)\binom{Q}{3}]\,(k - 1)\Big] \quad \text{(B30)}$$

$$a_2 = B\,[1 - \binom{Q}{3}(1 - \binom{P}{3})] + 6/k - \kappa[Q/2\,(B - 7k) - (3q - 1)(k - 1) + \tfrac{1}{2}(P - Q)\{4 + (B + 1)(1 - q)\}] - (1 - \kappa)\Big[\Big(P(B/2 + 2 + q) - Q\{B/2 + 1 - 4(1 + 4q)\}\Big)(2 - k) + \Big(\tfrac{1}{4}(B - 2)\{1 + 3/2(P - Q)\} - B/2\,(1 - q) - \binom{P}{2}[\{\tfrac{1}{2}(B + q - \varepsilon q_x) + 3\,\varepsilon q_x\}(2 - \varepsilon q_x) - \tfrac{1}{4}(B + 2)(1 - q)]\Big)(1 - \binom{Q}{3})(k - 1) - \binom{P}{3}[2\,(1 + \varepsilon q_x) + \tfrac{1}{2}(2 - q)\{3(1 - q) + \varepsilon q_x - q\} - q/4\,(1 - q)(B - 4) - \tfrac{1}{4}(B - 2) + B/2\,(1 - q)]\Big] \quad \text{(B29)}$$

$$a_3 = 4\,B\,y'/(y'+1) - (B + 4)^{-1} \quad \text{(B31)}$$

with

$$y'\,2\,B = \kappa[\sqrt{h}/k\,\{4\,(2 - \sqrt{\eta}) - \pi\,e\,(1 - \eta)\sqrt{h}\}\{k + e\sqrt{h}\,(k - 1)\} + \frac{5(1-q)}{2k+(-1)^k}(4B + P + Q)] + (1 - \kappa)\Big[(P - 1)(P - 2)\{2/k^2\,(H + 2) + (2-k)/(2\pi)\} + \binom{P}{2}(1 - \binom{Q}{3})\Big(q\,B/2\,\{B + 2(P - Q)\} + \Big\{P\,(P + 2)B + (P + 1)^2 - q(1 + \varepsilon q_x)\,[k(P^2 + 1)(B + 2) + \tfrac{1}{4}(P^2 + P + 1)] - q\,(1 - \varepsilon q_x)(B + P^2 + 1)\Big\}(k - 1) + \Big\{(P - Q)(H + 2) + P[5\,B\,(1 + q)\,Q + k\,(k - 1)\,\{k(P + Q)^2(H + 3k + 1)(1 - q) - \tfrac{1}{2}(B + 6k)\}]\Big\}(1 - \binom{P}{2})(1 - \binom{Q}{3}) + \binom{P}{3}(2 - q)\,Q\,\{\varepsilon q_x(B + 2Q + 1) + q/(2k)(1 - \varepsilon q_x)(2k + 1) + (1 - q)(Q^2 + 1 + 2B)\}\Big]$$

The calculated results for B29, B30, B31 and B22 can be found in a table VIII/chapter G.

For the excitation function f from eq.(XXXV) Heim got the expression

$$f\,(N) = a\,N/(N+1) + b\,N \quad \text{(B32)}$$

with the substitutions (α is the finestructure constant):

$$a = \frac{P^2}{kXh_{q,k}^2\sqrt{h_{q,k}}}(1 - k/4) + (k - 1)\{\pi/4\binom{P}{3} - \eta_{1,1}\eta_{1,2}\binom{P}{2}\} \quad \text{(B33)}$$

$$X = \kappa\Big[4\alpha\frac{(B+k+1)}{(1-a^2)}\frac{(1+5a^2)}{(1-5a^2)} - 2\left(\frac{3a}{4p}\right)^2 - q[p/2 - 1 - a^2\frac{pe(1+\sqrt{h})}{2J(1-6a^2)}\Big] + 1 \quad \text{(B34)}$$

$$b = \frac{1}{2h_{qk}^2\sqrt{h_{qk}}}\Big[\alpha J\ /8\,(P^2+1)[\,½\,(1+\sqrt{h}\,)(1+\eta_{1,1}\,\eta_{1,2}\ (3/4)\tbinom{P}{3}(k-1)] +$$
$$+ (k-1)\,\{J_{1,2}/J\ - 8\tbinom{P}{2}(P^2+1)^{-1}\}\Big] - C \qquad \text{(B35)}$$

$$C = \pi\,(1-\sqrt{h}\,)^2\Big[1+\sqrt{p}\,(k-1)+P/k^3\big(3/e+q(8+\eta_{qk})+$$
$$+ (4\,\pi e/\sqrt{h}\,)(1-\kappa)[1-q\,\frac{3ph}{5eh_{qk}}\,] - 2(k-1)\tbinom{P}{2}(3-P)\{2\,e\,(\eta+\eta_{qk}) \qquad \}\text{(B36)}$$
$$+\ \varepsilon q_x\,\pi e/(3\sqrt{h}\,)\,\} + \frac{8pek\,(k-1)}{\sqrt{h}}\left(\frac{e}{\sqrt{h}}-\frac{q}{\sqrt{e}}\right)\big)\Big] + (2\,e\,\kappa\,q/\eta^2)(2-k)(1-\eta)^2$$

The excitations can lead to a change of angular momentum. Since Q is the double quantum number of angular momentum, Q(N = 0) could change additive by an even number 2z with the integer function z(N), such that:

$$Q\,(N) = Q\,(N=0) + 2\,z\,(N), \qquad \text{(B37)}$$

where z(N) is yet unknown.

One has to hold in mind, that the σ-fillings of the external region of a term M(N) can get an additional excitation because of their external character. If the zones n_N, m_N, p_N, and σ_N are occupied and if

$$L_{(\sigma)}(p) = ½\,N_3\,(p+Q_p) - 2\,Q_\sigma \quad \text{with} \quad -Q_\sigma \le \sigma \le L_{(\sigma)}(p)\,, \qquad \text{(B38)}$$

is the complete occupation of the external region related to p_N , then

$$K_B = L_{(\sigma)}(p) - \sigma_N \qquad \text{(B39)}$$

describes a real number, which as a bandwidth determines the number of the possible excitations of the external field of an excitation state M(N). For $K_B \le 0$ there is no possibility of an external field excitation.

If $L_{(N)}$ describes the maximal occupation of all the four structure zones $0 \le N \le L_{(N)} < \infty$, then the equation of the excitation limit is given by eq.(XXXV) and eq.(B32) with $N = L_{(N)}$.

If the quantum numbers k, P, Q, κ, and q_x , as well as the excitation N, are given for a basic state, then the right-hand side of eq. (XXXV), i.e.

$$(n+Q_n)^3\alpha_1 + (m+Q_m)^2\,\alpha_2 + (p+Q_p)\,\alpha_3 + \exp[-(2k-1)/3Q_\sigma(\sigma+Q_\sigma)] =$$
$$= W_{N=0}(1+f(N)) \qquad \text{(B40)}$$

with $\alpha_1 = N_1$, $\alpha_2 = 3/2\,N_2$, $\alpha_3 = ½\,N_3$, and eq.(B22) to eq.(B36) can be calculated numerically.

By an exhaustion process based on

$$w = W_{N=0}(1 + f) \qquad \text{(B41)}$$

n, m, p, and σ can be determined using eq.(B15) to eq.(B21) and (B40).

Let be $K \geq 1$ the series of natural numbers. Then, first of all, $w - K^3\alpha_1 \geq 0$ will be formed. K will be raised as long as $K = K_n$ changes its sign. Then K_n will reduced by 1, which results in:

$$w - (K_n - 1)^3 \alpha_1 = w_1 \qquad \text{(B42)}$$

The process will be repeated with w_1 in the form $w_1 - K^2 \alpha_2 \geq 0$. With $K = K_m$

$$w_1 - (K_m - 1)^2 \alpha_2 = w_2 \qquad \text{(B43)}$$

will be generated. In the same way $w_2 - K \alpha_3 \geq 0$ yields the relation

$$w_2 - (K_p - 1) \alpha_3 = w_3 \qquad \text{(B44)}$$

and with the abbreviation $ß = (2k-1)/3Q_\sigma$

$$w_3 - e^{-ßK} \leq 0 \qquad \text{(B45)}$$

is determined, which changes its sign for $K = K_\sigma$. Next, K_σ will be reduced by 1.With the limits now known, K_n to K_σ , the n, m, p, σ can be calculated:

$$\begin{aligned} n &= K_n - 1 - Q_n \qquad & m &= K_m - 1 - Q_m \\ p &= K_p - 1 - Q_p \qquad & \sigma &= K_\sigma - 1 - Q_\sigma \end{aligned} \qquad \text{(B46)}$$

With these quantum numbers the mass formula (B3) with its parts eq.(B4) to eq.(B14) can be calculated.

2. The Average Life Times of the Basic States

Let be T the average life time of the masses of elementary particles determined by eq. (B3). If $T_N = T(N) << T$ is a function depending on N, so that $T_0 = 0$ for N = 0, then according to Heim the unified relation for the **times of existence** is:

$$(T - T_N) =$$

$$= \frac{192hHy}{Mc^2[h_{2,2}(1-\sqrt{h})^2(1-\sqrt{h_{1,1}})^2(1-\sqrt{h_{1,2}})^2](H+n+m+p+s)(n+|m|+|p|ß_{(0)})}\delta \qquad \text{(B47)}$$

where $\delta = \delta(N)$ is as in eq.(B7) .M is taken from eq.(B3), and H from eq.(B24). The substitution y is given by:

$$y = F\,[\varphi + (-1)^s (1+\varphi)(b_1 + b_2/W_{N=0})] \tag{B48}$$

with

$$\varphi = \frac{N_4 p^2}{1+p^2}\,\frac{s+Q_s}{\sqrt{1+s^2}}(\sqrt[4]{2} - 4BUW_{N=0}^{-1}) + P\,(P-2)^2(1+\kappa(1-q)/(2\alpha J))$$
$$(\pi/e)^2\sqrt{\eta_{12}}(Q_m - Q_n) -- (P+1)\binom{Q}{3}/\alpha\,, \tag{B49}$$

$$U = 2^Z\,[P^2 + 3/2\,(P-Q) + P(1-q) + 4\kappa B\,(1-Q)/(3-2q) + (k-1)\{P + 2Q -$$
$$- 4\pi(P-Q)(1-q)/\sqrt[4]{2}\,\}]\,/\eta_{qk}^2 \tag{B50}$$

and $$Z = k + P + Q + \kappa \tag{B51}$$

The calculated results for B48 and B49 can be found in a table IX.

B will be calculated from eq.(B28). It is(B52)

$$F = 1 - 1/3\,(1-q)(P-1)^2(3-P)(1 + P - Q - \varepsilon\,C\,P/2)(1 + ß_{(0)}(-1)^k) - \binom{P}{3}(1+D), \tag{B52}$$

$$s = 2 - k + \varepsilon\,C + (2kQ - \kappa P) + \binom{Q}{3} : 1/k\,(P-1)(P-2)(P-3) \tag{B53}$$

$$b_1 = [P\,\{\,7 + 6(1-q)(C - \binom{P}{2}) - 2q\,(1 - \binom{P}{2})\} + \kappa\,Q\{(3\,Z-1)\,B + 1\}]\,(2-k) +$$
$$+ \tfrac{1}{2}\,(1-\kappa)\{(q - \varepsilon q_x - 2)\,Q + \varepsilon\,C\,P + 2\,(P+1) - \quad \}(B54)$$
$$- (1-q)\,\frac{P(P-3)}{1+P(P^2-1)}\,(4\,B - 6 + P)\}(k-1) - \binom{P}{3}(q - \varepsilon\,q_x)$$

$$b_2 = B(5B+3) + \frac{2H-3}{P+1} + C^k\{B(\,3B+2(H+1)) + H + \tfrac{1}{2}\,\}(1-q) - Q\,\{B(2(B+H)-1) +$$
$$+ H/2 + 3\} + \kappa\,q\,\{B\,(3B+1) - 5/2\}(k-Q) - \binom{P}{2}P^2(P+Q)^2[8B+1 -$$
$$-\{5B - (2H+1)(1 + 2\binom{P}{3} - Q) + 2\}\,q] - \binom{P}{2}\,H(1-q) - (B - 3/4)^2(P-1)(P-2)(P-3)(-1)^{k-1} +$$
$$+ (Q-q)(1-q+Bq)\{3(H+B) + \pi e/\eta - q/4\}(P+1)^3(k-1) + \kappa\{(-1)^{1-q}\,[7HB + 3(H+B) - 5/2 +$$
$$+ (1-q)\{H(3B-4) + B + 7/2\}](k-1) + Q\binom{P}{2}\{(2-q)(1+\varepsilon\,q_x)[B/2(H+2) + 3/4\,] + 5/2HB +$$
$$+ 3H - \frac{B+5}{P+1}\,\} - 5/2\,H^2\binom{P}{3}\{q\,(1 + \pi/3(2-q)\,\eta_{2,2})\,B - (2-q)(1-q)\} \tag{B55}$$

with $$ß_{(0)} = \frac{2a}{pe}\left(\frac{1-\sqrt{h}}{1+\sqrt{h}}\right)^2 \tag{B56}$$

and $$D = [1 + 4\,q^2(q-1)(2q+1)]^{-1}\,\eta\,ß_{(0)}\,(1-\sqrt{\eta})^4\,P^{2+\varepsilon q}\,(P-1)^{(q-1)q/2}/(3\sqrt{2}) \tag{B57}$$

With the systems eq.(B3) to eq. (B14) and with the quantum numbers (Table I) the particular masses M can be calculated, and from eq.(B47) to eq.(B57) the life times T of all the multiplet components for N = 0 can be determined numerically and compared with empirical values (Table II). The life times T are shown in multiples of 10^{-8} seconds.

3. The Sommerfeld Finestructure Constant:

In φ and $ß_{(0)}$ the finestructure constant α is contained. The value in chapter D, section 8 is calculated only approximately. Heim now also gives the exact formula for α:

According to eq.(8.21) we get:

$$\alpha \sqrt{1-a^2} = \frac{9J}{(2p)^5}(1-C') \qquad \text{(B58)}$$

with
$$1 - C' = 1 - \frac{1+h_{2,2}}{h h_{1,1} h_{1,2}} \left(\frac{1-\sqrt{h}}{1+\sqrt{h}} \right)^2 = K_a \qquad \text{(B59)}$$

With the abbreviation

$$D' = \frac{(2p)^5}{9JK_a} \qquad \text{(B60)}$$

it follows for the reciprocal square of these solutions:

$$\alpha_{(\pm)}^{-2} = ½ D'^2 (1 \pm \sqrt{1-4/D'^2}) \qquad \text{(B61)}$$

With eq. (V/chapter E) the values for both branches are calculated:

$$\alpha_{+} = 0.72973525 \times 10^{-2} \quad \text{and} \quad \alpha_{-} = 0.99998589 \qquad \text{(B62)}$$

$1/\alpha_{(+)}$ = 137,03601	$1/\alpha_{( )}$ = 1,0000142

which, compared with the empirical value (Nistler & Weirauch 2002) for the finestructure constant,

$$1/\alpha_{(+)} = 137{,}0360114 \pm 3.4 \,.10^{-8}$$

yields a value which falls into the tolerance region of measurement. The negative branch shows an extremely strong interaction, which probably is based on the inner connections of the four zones in an elementary particle. But Heim did not investigate this further.

4. The Masses of Neutrino States

Supposing that in the central region of an elementary particle an euclidian metric rules, i.e. that there is no structure element, than that means: $L_{(n)} = -Q_n$.

According to eq.(B15) it means that there also is no ponderable mass M_0 . According to eq.(B16) to eq.(B21) it follows, that also the remaining structure zones are governed by an euclidian metric. In eq.(B3) then we must substitute

$$n = -Q_n ,\ m = -Q_m ,\ p = -Q_p \ \text{und}\ \sigma = -Q_\sigma , \qquad \text{(B63)}$$

from which follows:

$$G + F + S = \varphi \qquad \text{(B64)}$$

According to eq.(B49) generally $\varphi \neq 0$ holds, in spite of $\sigma + Q_\sigma = 0$, and also $\Phi \neq 0$ is not affected by the lower barrier of the n. m, p, σ . If $\Phi + \varphi \neq 0$, since $P > 0$ or $Q > 0$, then eq.(B49) yields a field mass unequal zero, in spite of eq.(B63). This field mass is not interpretable as a ponderable particle, but is - according to Heim - a kind of „spin-potence“ which as a „field catalyst“ permits transmutations of elementary particles or enforces the validity of certain conservation principles (angular momentum). This behaviour is equivalent to those properties which made the definition of neutrinos necessary by empirical reasons.

If according to eq.(B3) one substitutes for the mass of neutrinos in whole generality

$$M_\nu = \mu\alpha_+ (\Phi + \varphi_0) \qquad \text{(B65)}$$

where φ_0 relates eq.(B49) to the lower bounds of n, m, p, σ, than it follows, that M_ν is determined only by the quantum numbers k, κ, P, and Q .

For $M_\nu(kPQ\kappa) > 0$ the following possibilities result:

M_ν (1110) = M_ν (1111) and M_ν (1200) in the mesonic region, and
M_ν (2110) and M_ν (2111) in the baryonic region.

In addition there is another neutrino, which only transfers the angular momentum $Q = 1$ and which is required by the ß-transfer. For this neutrino only the two possibilities exist:

M_ν (2010) or M_ν (1010).

Since in the case (2010) $M_\nu < 0$ would be, only M_ν (1010) remains as a possibility for the ß-neutrino. With i = 1,...,5 the possible neutrino states ν_i are:

for k = 1: ν_1 (1010) , ν_2 (1110), ν_3(1200)
for k = 2: ν_4 (2110) , ν_5 (2111).

For each ν_i there exists the mirror-symmetrical anti-structure $\bar{n}_i$. From eq.(B3) with the possibly non-zero quantum numbers the neutrino-masses may be determined.

The calculated results are collected in table II. The masses are given in electron volt.

The empirical ß-neutrino can be interpreted by ν_1 and the empirical μ-neutrino by ν_2.
For the time being it cannot be decided whether the rest of the neutrinos also are implemented in nature or whether it concerns merely logical possibilities.

5. Concluding Remarks

For the numerical investigation of the states $N > 0$ the system (B32) must be used, which is uncertain because of the uncertain relations eq.(B33) to eq.(B36). The function z(N) in eq.(B37) must still be determined. Since z is not given, also Q(N) for N > 0 remains unknown. The mass values of the spectra N > 0 which belong to the basic states therefore still have an approximate character. Also the life times T_N of such states cannot be described yet. In eq.(B49) the free eligible parameters for the expression φ with eq.(B50) were fitted by empirical facts [i.e. $\sqrt[4]{2}, (p / e)^2$ and $4p\sqrt[4]{1/2}$] .

The error Q(N) = Q(0) = Q based on the approximation z = 0 for all of the N only causes an approximation error less than 0.1 MeV.

In spite of the mentioned uncertainties the numerical calculation of the relations eq.(B22) to eq.(B36) and eq.(B3) yields a spectrum of excitations for each basic state, whose limits are given by eq.(XXXV) with eq.(B32), and whose finestructure is described by eq.(B39).

In these spectra of excitation all empirical masses of short living resonances fit which were available to Heim at that time (CERN - Particle Properties - 1973). But there are much more theoretical excitation terms than were found empirically. That could be caused either by the existence of a yet unknown selection rule for N, or the selection rule is only pretended since the terms are not yet recordable by measurements.

In the tables IV and V Heim listed only such states N > 0 which seem to be identical with empirical resonances. The N-description in the third column differs between N and $\underline{N}$, where the underlining means that a term is addressed which does not fit the selection rule for N of the masses $M(N_B) - M(N_A) > 0$ with $N_B > N_A$. The values put in brackets in the 3rd and 4th column (with K_B from eq.(B39)) are related to possible electrically charged components. For the Δ -states, q = 2 was used. In the 5th column, the theoretical masses in MeV are indicated.

Here also the brackets are related to electrically charged components. The resonance states in general are represented very well, in spite of the approximate character (because ofz(N) = 0), but the uncertainty appears for k = 1 in the particles ω(783) and η'(958), as well as for k = 2 in the particle N(1688).

While the functions z(N) and T_N yet have been searched for by Heim, he already possessed an ansatz for a unified description of magnetic spin moments of particles with $Q \neq 0$, which was not yet published.

After discovering z and T_N, Heim wanted to calculate the cross sections of interaction, which regrettably could not more be done.

Apart from the above-mentioned incompleteness, it can be stated that on the basis of the far-reaching correspondence with the empirical data Heim's structure theory meets all requirements to be fulfilled by a mathematical scheme for a unified theory, and there is no other unified structure theory which allows for more exact or much better confirmed descriptions of the geometro-dynamical processes within the microregion.