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The Standard Model and quantum state reduction from Heim's field theory

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Abstract: Core parts of the Standard Model are derived from B. Heim's quantum field theory, whose poly-metric describes spacetime and matter in a unified formalism. Its non-linear eigenvalue equation transforms into the Einstein field equation in the macroscopic limit. The 6-dimensional Heim space can be determined as locally isomorphic to a $SU(2) \otimes SU(2) \otimes U(1) \otimes U(1)$ symmetry and thus to the $SU(3)$, which allows to connect to the local gauge symmetries and boson fields of the Standard Model. The Fermion and Higgs field and their coupling are deduced from Heim's basic equations, providing new insight into possible correlations of these fields. Furthermore, the derivation yields an additional imaginary coupling term which seems to account for quantum mechanical state reduction in the non-relativistic limit. The recently performed calculation of the mass spectrum of elementary particles in a new approach based on Heim's theory (with average error to the data <1%) appears as even more relevant, having now shown that the theory can connect to the achievements of the Standard Model.

Keywords: general relativity; particle masses; quantum field theory; quantum state reduction; standard model.

1 Introduction

While the Standard Model (SM) is still the accepted basic theory of particle physics, explaining a wide range of phenomena in high-energy physics [1], there have been attempts for decades to find a more fundamental and unified theory. This is due to the fact that the SM cannot explain some important questions, amongst others the cause of the generations of elementary particles (leptons, quarks) and the origin of their masses. As emphasised in [2], "the Higgs mechanism provides a model of breaking the electro-weak symmetry and attaching mass to particles, but it leaves the number of free parameters the same, as there is a Yukawa coupling constant for each mass to be determined" [3]. Furthermore, "the SM is a theory on a fixed Lorentz invariant 4-dimensional spacetime, leaving gravitation and general covariance completely out".[1] "The well-known candidates for a more unified and fundamental theory as GUTs, Supersymmetry, Superstring and M-theory are not (yet) able to give answers on the issue of the masses and possibly will never do it [3, 4]". Partner particles of Supersymmetry have not been detected so far.

In recent years various alternative approaches have been explored which also go beyond the SM and may provide new insight into the issues raised (e.g. [3, 5–8]). We have created a new model to calculate the mass spectrum of elementary particles based on the little known alternative quantum field theory of B. Heim and were able to reproduce the particle masses, including those of the muon and τ-lepton, with an error <1% on average. The obtained mass hierarchy levels are not identical to the particle generations of the Standard Model, however, show a self-similarity typical for non-linear theories [2]. The theory of Heim, which we shall outline in the next section, has some interesting properties which are important for a fundamental theory candidate: It is supposed to describe all matter in a unified way, and even more, by its geometrical approach, spacetime and matter in one model of a discretised space (which excludes problems with infinities from the start). It manifests in a non-linear eigenvalue equation, which merges into the Einstein field equation in the macroscopic limit. It also could be shown that the Heim equations contain the Dirac equation as a *possible* solution in the linear approximation [2, 9], which is in agreement with the linear nature of quantum mechanics.

As all approaches for a fundamental theory, also the Heim theory should reproduce the structure and properties of the Standard Model in respective limits if it shall be considered as a serious candidate for a "Theory of Everything".

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1 As is known, there are further limitations and problems regarding the SM, like the strong CP problem, the neutrino masses/oscillations, the hierarchy problem, matter-antimatter asymmetry, missing answers to dark matter and energy, etc. We have focused on the problems of the masses and generations, as they can be addressed towards a solution by means of Heim's theory, see [2].

Heim's theory, including our approach to calculate the particle masses, obviously can provide advancement compared to the SM regarding the origin of the masses and the unification with General Relativity (GR), but it must be clarified whether it can also connect to the achievements of the SM. Therefore, we analyse in this paper how this connection between Heim's theory and the formalism of the SM can look like.

This analysis will not only provide core parts of the SM, but also an additional imaginary term in the coupling between fermion and Higgs field which generates a non-unitary reduction, thus collapse of the wave function in the non-relativistic limit. We examine the properties of the obtained result and apply it to the measurement process with a macroscopic apparatus, but also to experiments of matter-wave interferometry with large molecular clusters, which can serve as experimental test.

2 Heim's quantum field theory

As already stated in [2], Burkhard Heim (1925–2001)[2] developed his theory over decades, but published it only once in a scientific journal in form of an overview article in 1977 [10]. Later, in the 80s, he released the details in two books [9, 11], but written in German language, which hindered the discussion of his theory. The present author, after an intensive study of the theory, recently published the above-mentioned work on the mass spectrum of elementary particles, based on Heim's theory, but with an own new ansatz for the calculation [2]. The quoted article contains a detailed summary of Heim's theory in English, so that it can be followed. We therefore refer to [2] for all details and present here only the core elements of the theory in form of a summary of Section 3 from [2], which is needed as input for our analysis in Section 3.

Heim's line of thought started with the equivalence principles between inert and heavy mass and mass and energy. Each particle with or without rest mass, through its energy, must be considered as a source of gravitation. Therefore, gravitation is the general background phenomenon which belongs to all particles as basis of matter. Thus, Heim takes the Einstein equation of GR as starting point, but introduces a quantisation through discretisation, since the physical action (and thus energy) must be quantised. This means for the left side of the Einstein-like equation, the geometric structure field, that it, and so spacetime, must be discrete. Heim derives a smallest discrete area which he calls *metron*.[3] From the discretisation of the structure field follows that the spacetime R_4 must be considered as a medium with a Hilbert functional space, i.e. a convergent state function (field) ϕ^i_{km} of the metric state of spacetime must exist and a hermitian state operator C_p, acting on ϕ^i_{km} in such a manner that an equivalent to the metric structure term arises

$$C_p\phi^i_{km} \rightarrow C_p\Gamma^i_{km} = R^i_{kmp} \tag{1}$$

with Γ^i_{km} being the Christoffel symbol and R^i_{kmp} the curvature tensor in the macroscopic realm.[4] On the other hand, because of the convergence of the state function and its hermiticity, this operator must define a spectrum of eigenvalues λ_p

$$C_{(p)}\phi^{(p)}_{km} = \lambda_{(p)}(k,m)\phi^{(p)}_{km} \tag{2}$$

(the brackets mean no summation over this index). These eigenvalues are proportional to energy densities, as the contracted macroscopic curvature tensor ($i = p$) is proportional to energy densities as well. They form a discrete point spectrum and give possible states of a microscopic field source. In this system of tensorial operator equations the 3 indexes run independently over the 4 spacetime dimensions, i.e. there are 64 discrete eigenvalue spectra of metric structure. As this system of eigenvalue equations is non-linear (due to C_p), the square of the absolute value of the $\phi^{(p)}_{km}$ cannot be interpreted as probability, since the solutions of the state functions of the metric structure do not additively superpose. So, the C_p describe non-linear metric states which occur in metric levels $\lambda_\mathrm{p}(k,m)$. On the other hand, these $\lambda_\mathrm{p}(k,m)$ are equivalent to the energy masses of those physical elementary structures which, according to Heim, appear as spacetime deformations in the sense of empirical elementary particles.

In the macroscopic realm the discrete energy density becomes a steady function $\rightarrow T_{ik} - \frac{1}{2}g_{ik}T$ and, with

2 Heim studied physics in Göttingen and got his diploma from C. F. v. Weizsäcker in 1954.

3 Metron $\tau = \frac{3}{8}\frac{\gamma h}{c^3}$ with γ being the gravitational constant, h Planck's constant and c the velocity of light [2]. This smallest area, which was also identified by Treder [12], is proportional to the square of the Planck length. Heim developed a whole new calculus with differences of these finite areas (see [11]). But for the physics we consider here, this metron theory need not be introduced, but normal calculus can be applied, because the number of metrons is so large that the spacetime quantisation can be neglected. Note that by means of the smallest area, similar as in Loop Quantum Gravity [13], the Bekenstein–Hawking entropy of a black hole can be derived as $S_\mathrm{BH} \sim \frac{A}{\tau} \sim \frac{c^3A}{\gamma h}$ with A being the surface area of the black hole, i.e. the event horizon.

4 Using the notation of Heim for the R^i_{kmp} which differs from the notation, e.g. in [14] by an exchange of the last two indices.

contraction of p and λ_{p} depending on k and m, $\lambda_{\mathrm{p}} = \lambda_{\mathrm{p}}(k,m)$, Eq. (2) turns into

$$C_p\phi^p_{km} \to R_{km} \quad \text{and} \quad \lambda_p\phi^p_{km} \to \kappa\left(T_{km} - \frac{1}{2}g_{km}T\right), \tag{3}$$

i.e. the Einstein equation is obtained again.[5] Because of the correspondence (1) the algebraic properties of the curvature tensor are transferred to $C_p\phi^i_{km}$, so from

$$R^i_{kmp} = \Gamma^i_{kp,m} - \Gamma^i_{km,p} + \Gamma^i_{ms}\Gamma^s_{kp} - \Gamma^i_{ps}\Gamma^s_{km} \Rightarrow$$
$$C_p\phi^i_{km} = \phi^i_{kp,m} - \phi^i_{km,p} + \phi^i_{ms}\phi^s_{kp} - \phi^i_{ps}\phi^s_{km} \tag{4}$$

follows, with $(), m := \partial_m = \partial/\partial x_m$. For $m = p$, $C_{(m)}\phi^i_{k(m)} = 0$ holds and therefore $\lambda_{(m)}\phi^i_{k(m)} = 0$ and thus

$$\lambda_m(k,m) = \lambda_m(m,k) = 0 \tag{5}$$

(the $\lambda_{\mathrm{p}}(k,m)$ are symmetric and real due to the hermiticity in (2)). So, 28 spectra are generally empty ($2\times 16 - 4$, the four spectra $m = k$ appear twice). The remaining $64 - 28 = 36$ discontinuous energy momentum densities $\lambda_{(p)}\phi^{(p)}_{km} \sim \epsilon^{(p)}_{km} \neq 0$ must be invariant against the allowed coordinate transformations, thus must be components of a 6-dimensional canonical energy momentum density tensor. With this line of thought Heim (in [11]) deduces that the R_4 should be extended by two hidden dimensions x_5 and x_6 to a R_6. He constructs the R_6 with a signature $I(+++---)$.[6] Different than in Kaluza Klein theories and in String theory the additional coordinates are not compactified, but get physical meaning. They cannot be two additional times, neither can they act on the movement of point particles in three-dimensional space. Therefore, Heim identifies these additional coordinates as parameters with organisational and informational effects which only act on structures, but not on points and their paths, thus influence the spacetime sector only via the time sequences T_{i4} and T_{4i} with $1 \le i \le 6$ and so only indirectly the R_3 (see Eq. (21) in [2]). As the coordinates x_5 and x_6 cannot (directly) be measured and the T_{ik} are microscopic quantities, this means that the future is always open in the microscopic realm, i.e. predications about the future can only be statements of probability. The extended Cartesian coordinate system then reads in Minkowskian notation (the $-$ signatures in η_{ik} lead to $\sqrt{-1} = i$ factors):

$$(x_1, x_2, x_3) \mathrel{\hat{=}} R_3,\ x_4 = ict,\ x_5 = i\epsilon,\ x_6 = i\eta \tag{6}$$

(6) suggests that the set of the R_6 coordinates is structured in the form $\{x_1, x_2, x_3\}$, $\{x_4\}$, $\{x_5, x_6\}$, i.e. in 3 partial structure units. The fundamental eigenvalue Eq. (2), according to Heim, now becomes valid for the R_6, i.e. with indices $p, k, m = 1, \dots, 6$ and the discrete eigenvalues $\lambda_{\mathrm{p}}(k,m)$ being proportional to energies. It describes all elementary structures of matter.

If in Eq. (2) "condensations" $\lambda_{\mathrm{p}}(k,m) \neq 0$ (as Heim calls it) do exist, then these $\lambda_{\mathrm{p}}(k,m)$ need not be defined in all coordinates of the R_6, but this spectrum can be related only to a subspace V_k with $1 \le k \le 6$, in which each metric tensor deviates from unity $g \neq E$. A physical interpretation of the k-dimensional deformable subspaces represents a *herme*neutics of the respective world geome*try*. Therefore, Heim uses the term *Hermetry* to distinguish that a deviation from Euclidean geometry is given in a V_k. If k coordinates are *hermetric*, then $6 - k$ Euclidean coordinates remain *anti-hermetric*. Heim analyses which combinations of the structure units are physically possible. He derives that the unit with x_5, x_6 must appear always as hermetric and so deduces the following 4 hermetry forms:

$$a \mathrel{\hat{=}} H(\{x_5, x_6\})$$
$$b \mathrel{\hat{=}} H(\{x_4\}, \{x_5, x_6\})$$
$$c \mathrel{\hat{=}} H(\{x_1, x_2, x_3\}, \{x_5, x_6\})$$
$$d \mathrel{\hat{=}} H(\{x_1, x_2, x_3\}, \{x_4\}, \{x_5, x_6\})$$

The forms a and b can be classified as imaginary condensations, the forms c and d as complex ones, as here also the real unit $\{x_1, x_2, x_3\}$ becomes hermetric. The form a denotes condensations in the trans-coordinates x_5, x_6, b time condensations, c space condensations and d spacetime condensations.

To solve the general hermetric problem, which we now write in component form using C_p from (4), the equation

$$\partial_l\left\{ {i \atop km} \right\} - \partial_m\left\{ {i \atop kl} \right\} + \left\{ {i \atop ls} \right\}\left\{ {s \atop km} \right\} - \left\{ {i \atop ms} \right\}\left\{ {s \atop kl} \right\}$$
$$= \lambda_m(k,l)\left\{ {i \atop kl} \right\} \tag{7}$$

5 Note that Heim's Eq. (2) is homogeneous and therefore (most likely) cannot predict the strengths, i.e. coupling constants of the interactions, including gravitation. This means that the structure of the Einstein equation is obtained again, but that the gravitational constant must be taken from the phenomenology. Another conclusion from the interconnections described above is that in Heim's theory a quantisation of GR by means of the canonical formalism is neither necessary nor appropriate. Heim's fundamental equation instead defines the quantisation in his theory and turns into the Einstein equation in the macroscopic realm.

6 The analogue in the R_4, $I(+++-)$, is defined by a metric tensor g_{ik} which, in an Euclidean or local geodesic space, becomes the diagonal η_{ik} with $\mathrm{d}s^2 = \eta_{ik}\mathrm{d}x_i\mathrm{d}x_k = \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2 - \mathrm{d}x_4^2 = \mathrm{d}r^2 - c^2\mathrm{d}t^2$. The $I(+++---)$ means in an analogous way $\mathrm{d}s^2 = \mathrm{d}r^2 - c^2\mathrm{d}t^2 - \mathrm{d}x_5^2 - \mathrm{d}x_6^2$ in a geodesic R_6 space.

has to be solved for the general situation of $1 < q \leq 6$ hermetric coordinates in the R_6. Heim here denotes the compositive state functions of the microscopic realm in the R_6, which converge to the macroscopic Christoffel symbols Γ^i_{km} of Riemannian geometry and GR, as $\left\{ {i \atop km} \right\}$ and calls them also composition field (to differentiate them from the state functions of the partial structures, see below). Based on symmetry relations between the $\lambda_m(k, l)$ and of the $\left\{ {i \atop km} \right\}$, one can sum over index m and transform Eq. (7) to

$$\left((a(k, l) - 1)\, \partial_l - \sum_{m \neq l} \partial_m \right) \phi_{kl} + \phi_{kl}^2 = \lambda(k, l) \phi_{kl} \qquad (8)$$

with the abbreviations and quantities $a(k, l) = \sum_{m=1}^{q} a_m(k, l)$, $b_i(k, l) = \sum_{m=1}^{q} b_{mi}(k, l)$, $\lambda(k, l) = \sum_{m=1}^{q} \lambda_m(k, l)$ and the function $\phi_{kl} = b_i(k, l) \left\{ {i \atop kl} \right\}$, all defined and derived in [2] (Appendix F). This is a Bernoulli differential equation which can be solved in a straightforward way, leading to a result expressed through the normalised function

$$\psi_{kl} = \frac{\phi_{kl}}{\lambda(k, l)} = \frac{b_i(k, l)}{\lambda(k, l)} \left\{ {i \atop kl} \right\} = \left(1 - \mathrm{e}^{-\vec{\lambda}_{kl} \vec{x}} \right)^{-1} \qquad (9)$$

where $\vec{x}$ is the 6-dimensional "position" vector $\vec{x} = \vec{r} + i\vec{\xi}$, $\vec{\xi} = (ct, \epsilon, \eta)$ and $\vec{\lambda}_{kl} = \frac{1}{q} \lambda(k, l)\, \vec{a}_{kl}^{-1}$ with $\vec{a}_{kl} = \vec{e}_l (a(k, l) - 1) - \sum_{m \neq l} \vec{e}_m$. From this solution also the metric tensor g_{ik} can be calculated (see [2], Appendix F).

We now come to a central aspect of Heim's theory, which was already heuristically introduced by the 3 partial structure units, to a "poly-metric" geometry. According to Heim, this poly-metric allows for a refinement of the Riemannian geometry so that internal structures (processes) of matter become describable on a geometric basis. The metric tensor g_{ik} is composed of a combination of partial structures. We can define poly-metric state functions, called "condensers" by Heim, depending on the partial metric structures, the squared brackets now denoting this composition by partial structures:

$$\begin{aligned} \left[{i \atop kl} \right] &= g^{is} \left[skl \right] = \frac{1}{2} \sum_{\kappa, \lambda = 1}^{3} g^{is}_{(\kappa\lambda)} \\ &\times \sum_{\mu, \nu = 1}^{3} \left(\frac{\partial g^{(\mu\nu)}_{ls}}{\partial x^k} + \frac{\partial g^{(\mu\nu)}_{ks}}{\partial x^l} - \frac{\partial g^{(\mu\nu)}_{lk}}{\partial x^s} \right) \\ &:= \sum_{\kappa, \lambda, \mu, \nu = 1}^{3} \left[{i \atop kl} \right]^{(\kappa\lambda)}_{(\mu\nu)} \end{aligned} \qquad (10)$$

In the abbreviated syntax the indices of the partial structures $(\kappa\lambda)$ denote the contravariant and the $(\mu\nu)$ the covariant basic signature.[7] Other than in Riemannian geometry, in which the product of co- and contravariant metric tensor gives the Kronecker symbol $g^{ik} g_{kj} = \delta^i_j$, in the polymetric theory this product in general is a function $f^i_j(\alpha) \neq \delta^i_j$ with $\alpha \hat{=} \left({\kappa\lambda \atop \mu\nu} \right)$ which expresses the correlation between the elements $(\kappa\lambda)$ and $(\mu\nu)$. This means that $\left[{i \atop kl} \right]$ in general becomes

$$\left[{i \atop kl} \right] = \sum_{\kappa, \lambda, \mu, \nu = 1}^{3} \left(\left[{i \atop kl} \right]^{(\kappa\lambda)}_{(\mu\nu)} + Q^i_m \left({\kappa\lambda \atop \mu\nu} \right) \left[{m \atop kl} \right]^{(\kappa\lambda)}_{(\mu\nu)} \right) \qquad (11)$$

with Q being a so-called correlation tensor which couples the respective partial structures. If g_{ik} consists of only one partial structure μ, then no correlation exists, i.e. $Q\left({\mu\mu \atop \mu\mu} \right) = 0$, as in the monometric Riemannian geometry of GR.[8] With the definition of the tensor

$$F^{(\kappa\lambda) i}_{(\mu\nu) kl} = \left[{i \atop kl} \right]^{(\kappa\lambda)}_{(\mu\nu)} + Q^i_m \left({\kappa\lambda \atop \mu\nu} \right) \left[{m \atop kl} \right]^{(\kappa\lambda)}_{(\mu\nu)} \qquad (12)$$

an insertion of expression (11) into Eq. (7) gives

$$\partial_l F^i_{km} - \partial_m F^i_{kl} + F^i_{sl} \left[{s \atop km} \right] - F^i_{sm} \left[{s \atop kl} \right] = \underline{\lambda}_m(k, l) F^i_{kl} \qquad (13)$$

after a term by term equation under the sum over the indices $\kappa, \lambda, \mu, \nu$ (here suppressed) for the general polymetric situation, with $\left[{i \atop kl} \right]$ as defined in (11), being the composition field. From this form immediately the same symmetry relations for the $\underline{\lambda}_m(k, l)$ follow as for the $\lambda_m(k, l)$ from Eq. (7) in the compositive case. With these relations and similar steps as in the solution of the composition field the F^i_{kl} can be calculated, see [2]. However, for our purposes Eq. (13) is sufficient and the right starting point for our development in Subsection 3.2.

7 The notation also expresses that the contravariant index i is provided by the partial structure κ. The index s of the partial structure λ facilitates the summation. As we see from (10) there are $3^4 = 81$ fundamental condensers in the poly-metric, instead of the Christoffel symbols in the Riemannian geometry, so a much richer overall structure.

8 Here we should emphasise that the poly-metric still contains two partial structures in the R_4, $\{x_1, x_2, x_3\}$ and $\{x_4\}$. But in the macroscopic realm of GR the r.h.s. of Eq. (2) becomes the macroscopic energy momentum tensor as shown in (3), which "forces" the l.h.s., i.e. the geometric side in (2) to behave as one metric structure.

3 Derivation of core parts of the Standard Model

As first step we should become aware of the properties that make up the Standard Model. In quantum field theory the Lagrangian densities set the basis for the theory and they depend on the fields which are part of it. In the SM these are the fermion fields, described by the quantised Dirac field, the gauge boson fields which appear through the local gauge symmetries $U(1), SU(2), SU(3)$ and the Higgs field which is coupled by Yukawa terms to the fermions and creates the mass terms of them and the heavy gauge bosons of the $U(1)/SU(2)$ electro-weak interaction (see e.g. [15, 16]). Quantum field theory is essentially "the only way to reconcile the principles of quantum mechanics with those of special relativity" ([17], Preface), for which reason Lorentz invariance will play a crucial role in the next subsection.

3.1 Gauge symmetries

The gauge symmetries entail the existence of internal degrees of freedom which are the weak hypercharge and isospin ($U(1) \otimes SU(2)$) and the colour ($SU(3)$). In Heim's theory first of all, these internal degrees of freedom are not defined, but the two additional dimensions x_5 and x_6.

To see whether Heim's theory can connect to the gauge symmetries, it is useful to consider first the properties of the theory in the limit of an approximately flat metric in the subspace R_4, i.e. for large distances (see Eq. (42)) compared to those very microscopic areas where the non-linearity of the theory dominates and the condensations (in Heim's words) "form" the particles. To recognise the r-dependent course and the symmetry of the metric in the R_4, the 4 hermetry forms have to be considered individually: According to Heim's analysis (see [2], Appendix G, and [11]), the a hermetry, anti-hermetric in the whole R_4 (see Section 2), generates gravity with discrete quantum levels (gravitons),[9] the gravitational field being locally Lorentz invariant. The b hermetry produces a quantised electromagnetic field (photons).[10] Here the R_3 coordinates occur together with the time coordinate as well-known term $\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$, so the Lorentz invariance is evident. In the c and the d hermetry instead, the R_3 coordinates are hermetric, which leads to r-dependent condenser functions F^i_{kl} and metric g_{ik}, but with a course so that $F^i_{kl} \to 0$ (see Eq. (53) in [2]) and $g_{ik} \to \eta_{ik}$, the Minkowskian metric,[11] for large r.

Thus, in the asymptotic flat metric the subspace R_4 locally follows the Lorentz group which again is locally isomorphic to $SU(2) \otimes SU(2)$ (see also [2], Appendix C.3). If now the dimensions x_5 and x_6 in general follow a $U(1)$ symmetry each (i.e. can be rotated in the complex plane, but without change of length), then the 6-dimensional space, consisting of the R_4 and the two trans-dimensions, should be locally isomorphic to $SU(2) \otimes SU(2) \otimes U(1) \otimes U(1)$. Furthermore, this overall symmetry matches that of the $SU(3)$ which can be seen by a Cartan decomposition of this eight-dimensional compact group [2, 18, 19]. Thus, we have shown the local isomorphism of the 6-dimensional Heim space to a duplicated structure of the $SU(2) \otimes U(1)$ and, via these, to the $SU(3)$ if the trans-dimensions x_5, x_6 follow a $U(1)$ symmetry and an asymptotically flat metric is given in the R_4. Then, such a space carries a symmetry which contains a $SU(2) \otimes U(1)$ as a subset, i.e. the gauge symmetry of the electro-weak interaction, and furthermore the $SU(3)$, the gauge symmetry of the QCD. To connect to the SM, we therefore must assume that the Heim space also, as a mapping, manifests in the stated internal degrees of freedom:

$R_6 \Rightarrow$ weak hypercharge, weak isospin, colour

Note that in the concrete formulation of Heim's theory (in Section 2) the dimensions x_5, x_6 are fixed as $x_5 = i\epsilon, x_6 = i\eta$, i.e. their above assumed $U(1)$ symmetry is broken. Therefore, it is consistent that the $SU(2) \otimes U(1)$ symmetry is broken in the SM as well, however, there not only the $U(1)$, but also the $SU(2)$ (see Subsection 3.3). This raises the question whether the $SU(2)$, that is, the $SU(2) \otimes SU(2)$, resulting from the Lorentz group of the asymptotic R_4 subspace, is broken under the conditions of Heim's full theory of the R_6, i.e. in the microscopic area, too. This is actually the case, since in the Heim space a Lorentz transformation does not generally preserve the metric locally, as it does in the flat R_4 of the SM in any case, but also in GR, where always a local reference system can be found in which the metric becomes Minkowskian, i.e. where each gravitational force can be transformed away. In the Heim space instead, the geodesic law

$$\ddot{x}^i + \begin{bmatrix} i \\ kl \end{bmatrix}^{(\kappa\lambda)}_{(\mu\nu)} \dot{x}^k \dot{x}^l = 0 \tag{14}$$

9 Although hermetry form a describes terms in x_5 and x_6 outside the R_4, it has an impact on the R_4 if x_5, x_6 depend on the coordinates of the R_4 in any form. This is the case if the structure a fulfils the condition of a null geodesic, because then the R_4-sector of the metric tensor is pseudo-Euclidian (due to its anti-hermetry) and the metric quantities of a, for $ds^2 = 0$, depend on the R_4-coordinates ([2], Appendix G).

10 See again [2], Appendix G. Because of the anti-hermetric R_3 coordinates, geodesic null lines exist $ds^2 = dr^2 + dx_4^2 = dr^2 - c^2t^2 = 0$ on the light cone.

11 If integration constants are set appropriately.

must hold for each condenser in the poly-metric. In fact, as explained in [2] (Subsection 3.5.3), for each element $(\mu\nu)$ of metric g a geodesic reference system can be found in which $\ddot{x}^i = 0$, but this property does not hold for each other element of g. So, if g consists of more than one structure unit, which is true for the hermetry forms b, c and d, no reference system can be found in which all condensations can be transformed away. Therefore, in the microscopic, not asymptotically flat realm of the R_6 the local Lorentz invariance does not hold in general and the $SU(2)$ as subalgebra of the Lorentz group is broken in Heim's theory.

The open question remains whether the $SU(3)$ symmetry then should be broken as well, since it is deduced via the above stated group theoretical relations from the doubled $SU(2) \otimes U(1)$ symmetry. But this seems to be the same question as why the $SU(3)$ is not broken in the SM, i.e. in the QCD. Another remaining question, namely how the above stated mapping "$R_6 \Rightarrow$ internal degrees of freedom" can be motivated more physically, not only mathematically, needs to be further investigated. (A possible approach could be that both sides have their origin in abstract quantum information bits, see [18].)

For our current analysis we consider the internal degrees of freedom and symmetries of the SM as being deduced from the isomorphisms above, as well as the symmetry breaking from the arguments given. We consequently apply the related rules including the known covariant derivatives etc. This generates already all gauge boson fields of the SM.

3.2 Fermion sector

Next the fermion fields shall be deduced. As onset we use an equation (Eq. (132) in [2]) which can be derived from (13) with similar steps as for Eq. (8):

$$\left((\underline{a}(k,l)-1)\partial_l - \sum_{m\neq l}\partial_m\right)F^i_{kl} + \underline{b}_s(k,l)\begin{bmatrix} s \\ kl \end{bmatrix}F^i_{kl} = \underline{\lambda}(k,l)F^i_{kl} \tag{15}$$

It is obtained after summing over the hermetric index $1 \leq m \leq q$ and by introducing analogous terms $\underline{a}(k,l)$, $\underline{b}_s(k,l)$ and $\underline{\lambda}(k,l)$ as in Eq. (8), defined in [2], Appendix H.2 (reference (131) et sqq.).[12] Next we use Eq. (9) with an expression for $\left\{ \begin{smallmatrix} i \\ kl \end{smallmatrix} \right\} (= \left[\begin{smallmatrix} i \\ kl \end{smallmatrix} \right])$ and the relation $\frac{1}{q}\sum_{s=1}^{q}\frac{\underline{b}_s(k,l)}{b_s(k,l)} = \frac{\underline{\lambda}(k,l)}{\lambda(k,l)}$, both from [2], Appendix H.2, which provides

$$\underline{b}_s(k,l)\begin{bmatrix} s \\ kl \end{bmatrix} = \frac{1}{q}\sum_s \underline{b}_s(k,l)b_s^{-1}(k,l)\lambda(k,l)$$
$$\times\left(1 - \mathrm{e}^{-\vec{\lambda}_{kl}\,\vec{x}}\right)^{-1} = \frac{\underline{\lambda}(k,l)}{\lambda(k,l)}\phi_{kl}. \tag{16}$$

With this expression and the vector $\vec{\underline{a}} = \vec{e}_l(\underline{a}(k,l)-1) - \sum_{m\neq l}\vec{e}_m$ (compare above) Eq. (15) can be rewritten to

$$\vec{\underline{a}} \cdot \vec{\nabla}_q F^i_{kl} = \underline{\lambda}(k,l)\left(1 - \frac{\phi_{kl}}{\lambda(k,l)}\right)F^i_{kl}. \tag{17}$$

We now use the same conversion as in [2] (reference (133) in Appendix H.2) by applying the inverse $\vec{\underline{a}}^{-1} = \vec{e}_l(\underline{a}(k,l)-1)^{-1} - \sum_{m\neq l}\vec{e}_m$ in the normalised orthogonal system of the q hermetric coordinates:[13]

$$\vec{\nabla}_q F^i_{kl} = \frac{\underline{\lambda}(k,l)}{q}\vec{\underline{a}}^{-1}\left(1 - \frac{\phi_{kl}}{\lambda(k,l)}\right)F^i_{kl} \tag{18}$$

Next we derive once again with the q-dimensional divergence $\vec{\nabla}_q$:

$$\vec{\nabla}_q \cdot \vec{\nabla}_q F^i_{kl} = \frac{\underline{\lambda}(k,l)}{q}\vec{\underline{a}}^{-1}$$
$$\times\left(\left(1 - \frac{\phi_{kl}}{\lambda(k,l)}\right)\vec{\nabla}_q F^i_{kl} - \left(\frac{\vec{\nabla}_q\phi_{kl}}{\lambda(k,l)}\right)F^i_{kl}\right) \tag{19}$$

The here appearing gradient $\vec{\nabla}_q\phi_{kl}$ can be calculated by deriving a similar equation as (18) for ϕ_{kl} from (8) with analogous steps

$$\vec{\nabla}_q\phi_{kl} = \frac{\vec{\underline{a}}^{-1}}{q}\left(\lambda(k,l)\phi_{kl} - \phi^2_{kl}\right) \tag{20}$$

12 Note that for simplicity of the notation we have again, as in reference (13), suppressed the indices of the partial structures in the expressions, but that the F^i_{kl} and the underlined terms depend on these indices.

13 The factor $1/q$ in Eq. (18) results from the fact that in the step from Eqs. (17) and (18) we fragment the scalar of (17) into q parts, which requires a division of the r.h.s. in (18) by q. It should also be noted that the step to Eq. (18) generates a certain ambiguity in the solutions of Eqs. (15), and (17). In [2] this was solved by inserting the result for F^i_{kl}, found by integrating (18), into (15) again to determine an appearing coefficient $(\underline{\alpha}_{kl})$ unambiguously, see Eq. (136) et sqq. in [2]. If this result is used to build the derivatives of F^i_{kl} in the following, the same result as in Eq. (23) is obtained, but with somewhat different expressions for the quantities $\underline{\eta}, \underline{\eta}'$ (in their dependence on the $a(k,l)$, $\underline{a}(k,l)$) in $\underline{\upsilon}$ and $\underline{\omega}$. As we will not be able to calculate these quantities from known data (see below), we refrain from specifying these slightly more complex expressions here and further follow the solution path solely with the differential equations as given below. It does not make any difference for our result structure.

so that the following result is obtained for the second derivative of F^i_{kl}:

$$\vec{\nabla}_q \cdot \vec{\nabla}_q F^i_{kl} = \frac{\underline{\lambda}^2(k,l)}{q^2}\underline{\vec{a}}^{-1} \cdot \underline{\vec{a}}^{-1}\left(1 - \frac{\phi_{kl}}{\lambda(k,l)}\right)^2 F^i_{kl} - \frac{\underline{\lambda}(k,l)}{q^2\lambda(k,l)}\underline{\vec{a}}^{-1} \cdot \vec{a}^{-1}\big(\lambda(k,l) - \phi_{kl}\big)\phi_{kl}F^i_{kl} \quad (21)$$

The products with the vectors $\underline{\vec{a}}^{-1}$ and $\vec{a}^{-1}$ can be calculated as

$$\frac{1}{q^2}\underline{\vec{a}}^{-1} \cdot \underline{\vec{a}}^{-1} = \frac{1}{q^2}\left(\frac{1}{(\underline{a}(k,l)-1)^2} + q - 1\right) := \underline{\eta}$$
$$\frac{1}{q^2}\underline{\vec{a}}^{-1} \cdot \vec{a}^{-1} = \frac{1}{q^2}\left(\frac{1}{(\underline{a}(k,l)-1)(a(k,l)-1)} + q - 1\right) := \underline{\eta}' \quad (22)$$

so that we get

$$\vec{\nabla}_q \cdot \vec{\nabla}_q F^i_{kl} = \frac{\underline{\lambda}^2}{\lambda^2}\left(\underline{\eta}\lambda^2 - 2\underline{\upsilon}\lambda\phi_{kl} + \underline{\omega}\,\phi^2_{kl}\right)F^i_{kl} \quad (23)$$

with the abbreviations $\lambda = \lambda(k,l)$, $\underline{\lambda} = \underline{\lambda}(k,l)$, $\underline{\upsilon} = \underline{\eta} + \frac{1}{2}\underline{\eta}'\frac{\lambda}{\underline{\lambda}}$ and $\underline{\omega} = \underline{\eta} + \underline{\eta}'\frac{\lambda}{\underline{\lambda}}$. The occurring quantities $a(k,l)$, $\underline{a}(k,l)$, λ, $\underline{\lambda}$ appear in the course of solving Heim's fundamental eigenvalue equations (for the composition field or a partial structure), see above, but cannot be calculated or directly related to calculable quantities (the particle masses) up to now (see [2]).

The result (23) is still universally applicable in the R_6. In order to derive a Dirac-like equation, we now have to approximate to the R_4, meaning that we assume a stationary status in which only the components of the R_4 are non-zero and all field components and also eigenvalues for the trans-dimensions x_5, x_6 become zero. In [2] (Appendix J) we have studied the linear limit of the Heim equations (according to [9]) and have shown that in case of the b hermetry, a linear approximation can formally lead to the known equations which determine the electromagnetic field and the particles interacting with this field, i.e. the equations of Maxwell in the classical limit and Dirac's equation for a relativistic fermion field (particle) as a *possible* solution in the linear limit. However, only its structure as linear partial differential equation of first order, acting on a complex 4-dimensional function, could be deduced unambiguously, but not its Clifford algebra, given by the γ matrices.

In the following we shall take a different path which starts with the general result (23) and does not explicitly limit the further development to the b hermetry. It keeps the non-linear term in the equations. In the R_4 the left side of (23) becomes the negative d'Alembert operator as

$$\vec{\nabla}_4 \cdot \vec{\nabla}_4 = \frac{\partial}{\partial(ict)}\left(\frac{\partial}{\partial(ict)}\right) + \sum_{j=1}^{3}\frac{\partial^2}{\partial x_j^2} = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \Delta = -\partial^\mu\partial_\mu, \quad (24)$$

which expresses the covariance to special relativity and is the known basis to derive a covariant equation with derivatives in first order, i.e. the Dirac equation. We therefore can reach our goal if we can also linearise the right side of (23) in a proper way. Before, we realise that the first two terms in the bracket on the r.h.s. of (23), i.e. terms constant or linear in ϕ_{kl}, result from the r.h.s. eigenvalue terms of Eq. (8) and (15). These represent the energy/mass of particle spectra which can be calculated directly from Heim's equations, see [2]. In keeping these terms in equations which shall lead to Lagrangians of the SM, one would put in results which still shall be derived by the terms responsible for the mass generation in the SM, i.e. the terms with the Higgs field. We therefore shall drop these terms of (23) in our final results, but keep them in the calculations before, in order to recognise their contributions. They will produce terms appearing with a negative sign, see $\underline{\gamma}_\alpha$ and $\underline{\delta}_\alpha$ in (25), which could lead to unphysical results. This step seems comparable to omitting mass terms of the Dirac equation in the electroweak Lagrangians of the SM while obtaining the correct mass terms later "dynamically" via the Higgs mechanism.

We now build the square addition to the r.h.s. of Eq. (23), fix the covariant indices k, l to 4 (index of the time coordinate in Heim's and our notation), i.e. consider only the equation for the energy (density) from the previous total set of Eq. (23), recognise that (23) does not differentiate in the index i, thus we can limit the further consideration to a fixed value of it and omit this index subsequently, but again explicitly note the dependency on the partial structure indices κ, λ, μ, ν, now summarised in the index α. Thus, we get the new equation

$$-\partial^\mu\partial_\mu F_\alpha = \frac{\underline{\lambda}^2_\alpha}{\lambda^2}\underline{\omega}_\alpha\left(\phi - \frac{\underline{\upsilon}_\alpha}{\underline{\omega}_\alpha}\lambda\right)^2 F_\alpha - \frac{\underline{\eta}'^2_\alpha\lambda^2}{4\underline{\omega}_\alpha}F_\alpha$$
$$:= \underline{\kappa}^2_\alpha\left(\phi - \underline{\gamma}_\alpha\lambda\right)^2 F_\alpha - \underline{\delta}_\alpha F_\alpha \quad (25)$$

valid in the R_4 (fixed indices $k, l = 4$ suppressed at all quantities). Note that we can build the sum $\sum_\alpha c_\alpha\times$ on both sides of Eq. (25) and get an equation for a superposition of F_α. The transformation to an equation in first order can be performed even for this superposition of F_α

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\Delta\right)\sum_\alpha c_\alpha F_\alpha = \sum_\alpha c_\alpha \underline{\kappa}_\alpha^2\left(\phi-\underline{\gamma}_\alpha\lambda\right)^2 F_\alpha - \sum_\alpha c_\alpha \underline{\delta}_\alpha F_\alpha \to \kappa^2\Phi^2\sum_\alpha c_\alpha F_\alpha \tag{26}$$

if

$$\kappa^2 = \frac{\sum_\alpha \underline{\kappa}_\alpha^2 c_\alpha F_\alpha}{\sum_\alpha c_\alpha F_\alpha}, \qquad \Phi = \phi - \gamma\lambda, \tag{27}$$

γ is an average value fulfilling the above relation expressed in (26), and the $\underline{\delta}_\alpha$ are approximately negligible, which seems to be the case,[14] and are, as γ, an omissible artefact as explained above. The quantity κ^2 can be identified as the mean of the $\underline{\kappa}_\alpha^2$ in the distribution given by the $c_\alpha F_\alpha$. If the superposition consists of a single term, $\kappa^2 = \underline{\kappa}_\alpha^2$ holds. So, with $F = \sum_\alpha c_\alpha F_\alpha$ we can derive the linear equation of first order in the derivatives

$$i\hbar\frac{\partial}{\partial t}F = \frac{\hbar c}{i}\left(\sum_{k=1}^{3}\alpha_k\frac{\partial}{\partial x^k}\right)F + \hbar c\,\beta\kappa\,\Phi F \tag{28}$$

while having already multiplied both sides with $\hbar c$. The terms on both sides of the equation in front of F have to be considered as operators acting on F. When they are iterated, the second order equation of (26) is reproduced again if the α_k, β fulfil the known anti-commutator relations and identities of the Dirac matrices. Thus, we have got a Dirac-like equation, i.e. a linear first-order differential equation in 4 spinor dimensions. The known form of the equation which explicitly shows the Lorentz invariance can be obtained by multiplying with the matrix β/c on both sides, introducing the gamma matrices $\gamma^0 = \beta, \gamma^i = \beta\alpha_i, i = 1,2,3$ and $\mu = 0,1,2,3$ [20] and by renaming the 4-dimensional spinor $F = \psi$:

$$\left(i\hbar\gamma^\mu\frac{\partial}{\partial x^\mu} - \hbar\kappa\,\Phi\right)\psi = 0 \tag{29}$$

This equation contains a mass-like term with the correct sign, which includes a coupling to a further field $\Phi = \phi - \gamma\lambda$, where $\phi = \phi_{k,l=4}$ directly relates to Heim's composition field, as defined by Eqs. (7)–(9). So, $\Phi = \Phi_R + i\Phi_I$ is a complex quantity. We now have to create a Lagrangian which reproduces the differential equation above (via the Euler–Lagrange equations), but also connects to the hermitian Lagrangian of the fermion sector in the SM. We therefore split the term containing Φ into its real and imaginary (hermitian and anti-hermitian) part and get:[15]

$$L_f = \overline{\psi}\,i\hbar\gamma^\mu\frac{\partial}{\partial x^\mu}\psi - \hbar\kappa\,\overline{\psi}\Phi_R\psi - i\hbar\kappa\,\overline{\psi}\Phi_I\psi \tag{30}$$

The term resulting from the real part $\Phi_R = (\Phi + \Phi^*)/2$ appears as already very similar to the Yukawa coupling term in the Glashow–Salam–Weinberg (GSW) theory, although the weak isospin and related left/right-handed fermions are not yet expressed.[16] We shall continue with it in Subsection 3.4. The imaginary term does not exist in the SM, so is either an unphysical artefact, or might represent physics beyond the SM. We shall come back to this issue in Section 4, after we have clarified the nature of the field ϕ, and thus Φ, in the next subsection.

But before, it still should be noted that Eqs. (26) and (27) form a system of coupled equations which in principle could be solved in a self-consistent iterated procedure (compare e.g. the Hartree–Fock method) by inserting the possible functions F_α and optimising the parameters c_α. However, it is not clear for the time being how these condenser class functions of Heim's formalism, derived in the R_6 (see [2], Appendix H), are to be transformed to the R_4 without losing essential structure. This step is excluded with the spinor Eq. (28) (and thus (29)) in any case, since the spinor $F = \psi$ constitutes a completely different quantity than the F_α. This means that the expression for κ^2 in (27) cannot be used for practical purposes, as the $c_\alpha F_\alpha$ remain unknown, as well as the other quantities occurring in the $\underline{\kappa}_\alpha^2$, as already stated.

3.3 Higgs field

The scalar field $\phi = \phi_{k,l=4}$ is defined as $\phi = b_i(4,4)\left\{ {i \atop 44} \right\}$ (with sum over index i; see Section 2), i.e. as contraction of the composition field $\left\{ {i \atop 44} \right\}$ with the term $b_i(4,4)$ which in turn is composed of the eigenvalues $\lambda_m(4,4)$ [2]. Thus, it contains the whole physics of spacetime and matter,

14 The order of magnitude of the $\underline{\delta}_\alpha$ can be seen if $\underline{\eta}'_\alpha \approx \underline{\eta}_\alpha$ and $\underline{\lambda}_\alpha \approx \lambda$ is assumed, then $\underline{\delta}_\alpha \approx \underline{\eta}_\alpha\lambda^2/8$. The leading first term on the r.h.s. of (25) instead becomes $\approx 2\underline{\eta}_\alpha\lambda^2$ if $\gamma_\alpha \to 0$ and $\phi = \lambda$, i.e. is about 16 times bigger.

15 We have assumed κ as real >0, see Subsection 3.4, Eq. (40).

16 In the GSW theory the Yukawa coupling term consists by construction of a term $-c_f\overline{\psi}_R\Phi^\dagger\binom{\nu_L}{\psi_L}$+ its hermitian conjugate.

considering the energy ($k, l = 4$), in Heim's theory. This field fulfills Eq. (20) and we therefore can make an analogous step as for the F^i_{kl} and calculate the second derivative (already in the R_4, neglecting x_5 and x_6) as

$$\begin{aligned}\vec{\nabla}_4 \cdot \vec{\nabla}_4\phi = -\partial^\mu\partial_\mu\phi = \frac{\vec{a}^{-1}}{q}(\lambda - 2\phi)\vec{\nabla}_4\phi \\ = \frac{\vec{a}^{-1}\cdot\vec{a}^{-1}}{q^2}(\lambda - 2\phi)(\lambda-\phi)\phi \\ = \eta(\lambda^2\phi - 3\lambda\phi^2 + 2\phi^3) \end{aligned} \tag{31}$$

by inserting (20) again and with η in the same form as $\underline{\eta}$ in (22), but with quantity a instead of $\underline{a}$.[17] The r.h.s. in (31) can be brought into a more symmetric shape by substituting with $\phi = \phi' + \lambda/2$ which leads to

$$\partial^\mu\partial_\mu\phi' + \eta\left(-\frac{\lambda^2}{2}\phi' + 2\phi'^3\right) = 0. \tag{32}$$

To compare to the SM we need to specify a Lagrangian which reproduces Eq. (32) and its complex conjugate, i.e. has a suitable symmetry. The following expression fulfills this requirement, except for the term with ϕ'^3 which cannot be symmetric in ϕ', ϕ'^*. We therefore set

$$\begin{aligned}L_{\phi'} = (\partial^\mu\phi'^*)(\partial_\mu\phi') - \eta\left(-\frac{\lambda^2}{2}\phi'^*\phi' + (\phi'^*\phi')^2\right) \\ = T(\phi') - V(\phi')\end{aligned} \tag{33}$$

which can be considered as the only appropriate form for the Lagrangian, reproducing (32) closely. This Lagrangian is identical to the one of the Higgs field in the SM (before inserting the gauge covariant derivatives, see [15]) with potential[18]

$$V(\phi') = \mu^2\phi'^*\phi' + \lambda_H(\phi'^*\phi')^2 \tag{34}$$

and $\mu^2 < 0, \lambda_H > 0$ if we set $\lambda_H = \eta$ and $\mu^2 = -\frac{\eta\lambda^2}{2}$. The aforementioned value conditions for μ and λ_H are ensured as always $\eta > 0$. So, with (33) we have got the right Lagrangian of the Higgs field, which means that the composition field of Heim's theory $\left\{ {i \atop 44} \right\}$ provides the Higgs field ϕ' via the relation

$$\phi' = \phi - \frac{\lambda}{2} = b_i(4,4)\left\{ {i \atop 44} \right\} - \frac{\lambda}{2}. \tag{35}$$

The spontaneous symmetry breaking manifests in the local minima of the potential V at $\phi'_{\min} \neq 0$. If we restrict ϕ' to real values, we find $\partial V/\partial\phi' = \eta(4\phi'^3 - \lambda^2\phi') = 0 \Rightarrow \phi'_{\min} = \pm\lambda/2$. ($\phi' = 0$ instead is a local maximum.) Inserting these minima of ϕ' into Eq. (35) gives $\phi = \phi' + \lambda/2 = \lambda$ or $= 0$, which is identical to the limits of ϕ_{kl} according to Eq. (9). Note that in [2, 11] the discrete eigenvalues which represent the particle mass spectrum were defined by the extrema $\psi_{kl} = 1$, see Eq. (9), so that $\phi = \lambda$ holds in this case. Thus, the particle masses obviously correspond to a value of the Higgs field of $\phi' = \lambda/2$.

We now follow the usual procedure to explicitly break the symmetry [15] and assume for the nonvanishing vacuum expectation value of the Higgs field in the two-dimensional space of the weak isospin

$$\begin{aligned}<\phi'>= \frac{1}{\sqrt{2}}\binom{0}{v},\ v^2 = -\frac{\mu^2}{\lambda_H} = \frac{\lambda^2}{2}, <\phi'>= \binom{0}{\frac{\lambda}{2}} \\ \text{and get } <\phi>=<\phi' + \lambda/2>= \binom{0}{\lambda}\end{aligned} \tag{36}$$

for field ϕ, occurring via $\Phi = \phi - \gamma\lambda$ (27) in our coupling term. This means $<\Phi>= (1-\gamma)\lambda\binom{0}{1}$.

Remember that this breaking of the weak $SU(2)$ symmetry is suggested by our considerations of Subsection 3.1, but note that the concrete implementation $<\phi'>=\sim\binom{0}{v}$ is a replication of the respective step in the SM.

These results for the fermion and Higgs fields, the coupling between them still put back for a moment, reproduce the respective terms of the SM. With the approach for the gauge symmetries to "emboss" the needed internal degrees of freedom as outlined in Subsection 3.1, the parts of the overall Lagrangian which result from the minimal coupling via the covariant derivatives and from the gauge boson vector fields can be obtained one to one, including the terms of the covariant derivatives of the Higgs field which create the masses of the heavy gauge bosons. We refrain from writing down these parts of the Lagrangian here, they are given in the textbooks [15, 16]. Thus, the term which obviously corresponds to the Yukawa coupling between fermion and Higgs field in Eq. (30) with $\Phi = \phi - \gamma\lambda = \phi' + \lambda/2 - \gamma\lambda$ remains to be analysed.

3.4 Yukawa coupling

To treat the hermitian Yukawa coupling term of the Lagrangian in (30)

$$L_Y = -\hbar\kappa\,\overline{\psi}\Phi_R\psi = -\frac{\hbar\kappa}{2}(\overline{\psi}\Phi\psi + \text{h.c.}) \tag{37}$$

17 The same result as in (31) is obtained by calculating $\vec{\nabla}_4 \cdot \vec{\nabla}_4\phi$ with the solution for ϕ as given in (9).

18 We have marked the field with a prime for consistency with our notation, other than in the textbooks, and the second prefactor with index H to avoid a confusion with our quantity λ.

it must be supplemented by the weak isospin degree of freedom in order to connect to the electro-weak theory. As stated in Subsection 3.1, we currently can only detect the overall isomorphism between the 6-dimensional Heim space and the $SU(2) \otimes U(1)$ and $SU(3)$ gauge symmetries, and therefore cannot deduce the concrete implementation with a doublet of left-handed fields/particles at this point, but must adopt it from the phenomenology, as in the SM. The term then becomes

$$\begin{aligned} L_{\mathrm{Y}} &= -\frac{\hbar\kappa}{2}\,(\overline{\psi}\Phi\psi + \mathrm{h.c.}) \\ &\to -G_f\left(\overline{\psi}_R\Phi^\dagger\begin{pmatrix}\nu_L\\ \psi_L\end{pmatrix} + (\overline{\nu}_L,\overline{\psi}_L)\Phi\psi_R\right) \end{aligned} \tag{38}$$

where we have named the prefactor $\hbar\kappa/2 = G_f$ in analogy to the notation in [15]. Inserting the value of Φ obtained from the symmetry breaking in (36) gives

$$\begin{aligned} L_Y &= -G_f(1-\gamma)\lambda\left(\overline{\psi}_R\psi_L + \overline{\psi}_L\psi_R\right) \\ &= -\frac{\hbar}{2}\kappa(1-\gamma)\lambda\,\overline{\psi}\psi \end{aligned} \tag{39}$$

and thus $m_{\mathrm{f}}c = \hbar\kappa(1-\gamma)\lambda/2$ for the acquired mass of the fermion particle. We now set $\gamma \to 0$, as reasoned in Subsection 3.2, insert the formal (but for practical purposes not usable) expression for κ from (27) and finally get

$$m_{\mathrm{f}} = \frac{\hbar}{2c}\sqrt{\frac{\sum_\alpha c_\alpha F_\alpha\left(\underline{\eta}_\alpha\underline{\lambda}_\alpha^2 + \underline{\eta}'_\alpha\underline{\lambda}_\alpha\lambda\right)}{\sum_\alpha c_\alpha F_\alpha}}. \tag{40}$$

To ensure real mass values the expression under the square root must be real and positive, i.e. especially $\underline{\eta}'_\alpha\underline{\lambda}_\alpha\lambda > -\underline{\eta}_\alpha\underline{\lambda}_\alpha^2 := -k^2$ (since $\underline{\eta}_\alpha > 0$).

Formula (40) means that the fermion mass derived this way is attributed solely to quantities of Heim's basic formalism, although these currently cannot be calculated from empirically known quantities, but the masses only via the mechanism developed in [2]. Moreover, the expression (40) must be considered as only a rough approximation compared to the solution presented in [2]: From Eq. (60) in [2] can be seen that the mass-energies derived there go linearly with $\lambda = \lambda(k, l = 4)$, in Eq. (40) instead with $\sqrt{\lambda + C}$. The derivations of the last 3 subsections indeed do only approximate Heim's full non-linear equations, see also the discussion in Section 5.

4 Beyond the Standard Model – imaginary term and state reduction

The approach, presented in [2], is clearly to be understood as outside the SM, as it calculates the particle mass spectrum from Heim's basic theory. Here, in our derivation of the SM from this basic theory, we have obtained an imaginary term which does not appear in the SM. But we shall nonetheless analyse its physical effect, as we want to take every term seriously and into account which results from Heim's theory. Thus, this term

$$L_{\mathrm{I}} = -i\hbar\kappa\,\overline{\psi}\Phi_{\mathrm{I}}\psi = -i\hbar\kappa\,\overline{\psi}\phi_{\mathrm{I}}\psi \tag{41}$$

remains to be examined, with ϕ_{I} as the imaginary part of field ϕ. Before going into details, it seems useful to consider the role of such an imaginary term from a general perspective. As said, in the SM such a term has no correspondence, particle interactions including annihilation, which might by associated with an imaginary term, are generated by hermitian coupling terms between the fields. There have been individual approaches to introduce an imaginary part into a Lagrangian, see e.g. [21],[19] but not to describe "normal" particle physics through it (as far as we know).

In Heim's theory particle interactions cannot yet be modeled. To connect its central "tools", the hermetry forms and condenser classes (see Section 2 and [2]), directly to particle processes seems to be a long way still, probably impossible. However, the theory contains an imaginary part in its fundamental equations, since the composition field and also the condenser functions of the partial metric structures are complex functions in general. Thus, also the expression to calculate the energy/mass of the spacetime "condensations" which Heim identifies as the elementary particles, first of all has an imaginary part, see [2], Eq. (57), and [9]. Heim concluded that the mass must be given by the real part, the imaginary part instead should represent an energetic fluctuation width and so, due to the quantum complementarity, the lifetime of the respective particle [2, 9]. But, lifetimes

19 Nielsen's and Ninomiya's attempt was to "unify usual equation of motion laws of nature with laws about initial conditions, second law of thermodynamics, cosmology". They saw the Higgs particle as "most promising for seeing the initial condition determining effects from the imaginary part" and predicted "that the width of the Higgs particle shall likely turn out to be (appreciably perhaps) broader than calculated by summing usual decay rates". The later detection of the Higgs in the LHC made this discussion basically obsolete.

calculated from our model do not meet empirical reality (see [2], page 746, footnote 33). We conclude that this imaginary part obviously must not be interpreted as the quantity which generates the decay of particles, but that these are governed through the known interactions, which are well described by the SM, and to which we connect through the shown link to the SM.

If the imaginary term is not responsible for particle decay in quantum field theory, then alternatively it could play a role in the non-relativistic regime of quantum mechanics (QM).[20] Although, as we shall see, its role in the very microscopic region will remain unclear, where the condensations in the R_6 make up the particles in Heim's theory, it seems possible that this imaginary quantity ϕ_I plays the role of an ubiquitous "background" field which affects every particle wave function. According to our derivation of the last section, it is, up to a constant, identical to the imaginary part of the Higgs field ϕ', see Eq. (35).

It is often non-trivial to recognise whether quantum field theory or non-relativistic QM are best suited to explain certain phenomena. This can be seen from the physics of macroscopic quantum systems (like crystals) [22] and the related question whether decoherence as reason for the *appearance of a classical world in quantum theory* [23] is the dominating mechanism, or field quanta as collective modes which create robust order over macroscopic distances, as it is the case in a crystal. According to [22] (Subsection 4.5.1) decoherence is "a useful criterion to scan the border between QM and QFT". In this sense we subsequently consider the non-relativistic realm of single particles and molecules, for which decoherence plays an important role if they are in contact with an environment, and therefore normal QM should be the right description.

For that it is useful to analyse the concrete form of field $\phi = \lambda(1 - \mathrm{e}^{-\vec{\lambda}\vec{x}})^{-1}$ according to (9) (covariant indices $k, l = 4$ suppressed). We can evaluate its real and imaginary part with use of the abbreviation $R = 1 - 2\mathrm{e}^{-\vec{\lambda}\vec{r}}\cos(\vec{\lambda}\vec{\xi}) + \mathrm{e}^{-2\vec{\lambda}\vec{r}}$ as

$$\begin{aligned}\phi &= \lambda(1 - \mathrm{e}^{-\vec{\lambda}\vec{r}}\cos(\vec{\lambda}\vec{\xi}) - i\,\mathrm{e}^{-\vec{\lambda}\vec{r}}\sin(\vec{\lambda}\vec{\xi}))/R \\ &\to \lambda(\theta - \mathrm{e}^{-|\vec{\lambda}\vec{r}|}\cos(\lambda_t ct) - i\mathrm{e}^{-|\vec{\lambda}\vec{r}|}\sin(\lambda_t ct)) \\ &\text{for } |\vec{\lambda}\vec{r}| > 1 \text{ and } x_5, x_6 \to 0 \end{aligned} \tag{42}$$

with $\theta = 1$ for $\vec{\lambda}\vec{r} > 0$, $\theta = 0$ for $\vec{\lambda}\vec{r} < 0$ and λ_t being the x_4-component of $\vec{\lambda}$. We consider the field in the outer region $|\vec{\lambda}\vec{r}| > 1$, because we can expect only this region to (approximately) follow the rules of linear QM.

4.1 Density matrix

We now want to see which effect the imaginary term L_I creates in the time development of a fermion particle. Since we cannot relate this term to known physics, we cannot either make any assumptions how the internal degrees of freedom might be involved in it and therefore neglect them in this consideration, which corresponds to $x_5, x_6 \to 0$. With use of (39) and $m_\mathrm{f}c = \hbar\kappa\lambda/2$ ($\gamma \to 0$), of (41), (42) and the definition $\Gamma(\vec{r}, t) = \mathrm{e}^{-|\vec{\lambda}\vec{r}|}\sin(\lambda_t ct)$ we can write the fermionic Lagrangian of (30) with non-zero imaginary part in ϕ as

$$\begin{aligned} L_f &= \overline{\psi}\left(i\hbar\gamma^\mu\frac{\partial}{\partial x^\mu} - m_\mathrm{f}c + i\,2m_\mathrm{f}c\,\Gamma\right)\psi \\ &\Rightarrow \left(i\hbar\gamma^\mu\frac{\partial}{\partial x^\mu} - m_\mathrm{f}c + i\,2m_\mathrm{f}c\,\Gamma\right)\psi = 0. \end{aligned} \tag{43}$$

We now build the non-relativistic limit in the usual way (see [20]) and get for the non-relativistic wave function Ψ

$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m_\mathrm{f}}\Delta - i\,2m_\mathrm{f}c^2\Gamma\,\Psi = H\Psi - i\,2m_\mathrm{f}c^2\Gamma\,\Psi. \tag{44}$$

From (44) and its dual equation a master equation for the density operator $\rho = \sum_n p_n|\Psi_n\rangle\langle\Psi_n|$ is obtained

$$\left\langle \vec{r}\,'\middle|\frac{\partial}{\partial t}\rho\middle|\vec{r}\right\rangle = -\frac{i}{\hbar}\left\langle\vec{r}\,'\middle|[H,\rho]\middle|\vec{r}\right\rangle - \frac{2m_\mathrm{f}c^2}{\hbar}\left\langle\vec{r}\,'\middle|\{\Gamma,\rho\}\middle|\vec{r}\right\rangle \tag{45}$$

in the position representation, with {} denoting the anti-commutator. The second term on the r.h.s. is non-unitary, i.e. reduces the density and can be further evaluated by

$$\begin{aligned}\left\langle\vec{r}\,'\middle|\{\Gamma,\rho\}\middle|\vec{r}\right\rangle &= \Gamma(\vec{r}\,', t)\left\langle\vec{r}\,'\middle|\rho\middle|\vec{r}\right\rangle + \left\langle\vec{r}\,'\middle|\rho\middle|\vec{r}\right\rangle\Gamma(\vec{r}, t) \\ &= 2\sin(\lambda_t ct)\,\mathrm{e}^{-(|\vec{\lambda}\vec{r}| + |\vec{\lambda}\vec{r}\,'|)/2} \\ &\quad\times\cosh((|\vec{\lambda}\vec{r}| - |\vec{\lambda}\vec{r}\,'|)/2)\left\langle\vec{r}\,'\middle|\rho\middle|\vec{r}\right\rangle \\ &\to 2\sin(\lambda_t ct)\,\mathrm{e}^{-|\vec{\lambda}\vec{R}|/2}\cosh(\vec{\lambda}\vec{\Delta}r/2)\left\langle\vec{r}\,'\middle|\rho\middle|\vec{r}\right\rangle. \end{aligned} \tag{46}$$

If $\vec{\lambda}\vec{r}$ and $\vec{\lambda}\vec{r}\,'$ are both >0 or both <0, then the result can be simplified to the last expression with $\vec{R} = \vec{r} + \vec{r}\,'$ and $\vec{\Delta}r = \vec{r} - \vec{r}\,'$. We now can derive an expression for the time dependency of the density $\left\langle\vec{r}\,'\middle|\rho\middle|\vec{r}\right\rangle := \rho(\vec{r}, \vec{r}\,', t)$

20 Heim's theory itself does not predetermine which "level" of the established theory system is best suited to take up the properties and terms of Heim's model.

if we neglect the unitary term and consider only the non-unitary:

$$\frac{\partial}{\partial t}\rho(\vec{r},\vec{r}\,',t) = -\frac{4m_f c^2}{\hbar}\sin(\lambda_t ct)\,e^{-|\vec{\lambda}\vec{R}|/2} \times \cosh(\vec{\lambda}\vec{\Delta r}/2)\rho(\vec{r},\vec{r}\,',t)$$
$$\Rightarrow \rho(\vec{r},\vec{r}\,',t) = \rho(t=0)\exp\left(-\frac{4m_f c}{\hbar\lambda_t} \times e^{-|\vec{\lambda}\vec{R}|/2}\cosh(\vec{\lambda}\vec{\Delta r}/2)(1-\cos(\lambda_t ct))\right) \quad (47)$$

A numerical evaluation becomes possible if we consider this result at the eigenvalues found in [2] for the elementary particle mass spectrum, i.e. at $\vec{\lambda}\vec{x} = \vec{\lambda}(\vec{r}+i\vec{\xi}) = (d+i)K(n)$ with the $K(n) = \frac{\pi}{2}(2n+1)$ or $= n\pi$ and parameter d as defined in [2], which gives

$$\Gamma \to e^{-K(n)d}\sin K(n)$$
$$\to e^{-K(n)d}(-1)^n \text{ for } K(n) = \frac{\pi}{2}(2n+1) \text{ and}$$
$$\to 0 \text{ for } K(n) = n\pi. \quad (48)$$

This result means that Γ is exactly zero for all integer eigenvalues (as multiples of π) of the mass spectrum, but $\neq 0$ for the half-integer eigenvalues, which describe i.a. the leptons e, μ, τ and the "basic" hadrons, the (constituent) quarks u, d, s and the nucleon in our model (see Table 1 in [2]), i.e. fermions.[21] From the relation $\vec{\lambda}\vec{r} = K(n)d$ (already used in (48)) and $|\vec{\lambda}\vec{r}| \leq |\vec{\lambda}||\vec{r}|$ we can estimate $|\vec{\lambda}| \geq K(n)d/|\vec{r}|$ and so calculate an approximate value for the $|\vec{\lambda}|$ [22] in dependence of the parameters given for the respective particles if we use the empirical radius of the particle. On this basis and with the relations $|\vec{\lambda}\vec{R}| \leq |\vec{\lambda}|R$ and $|\vec{\lambda}\vec{\Delta r}| \leq |\vec{\lambda}|\Delta r$ we calculate approximate values for the argument of the exp() and the density ρ in Eq. (47), see Table 1. We want to consider normal matter consisting of atoms, i.e. nuclei and electrons, and as electrons have negligible mass compared to nucleons (and no empirical radius is known for this still point particle), we only consider the nucleons, which we do by using the parameters of the proton from [1, 2] (neutrons would yield only negligibly different results in our estimation). Beside scenarios for a single proton with two different values for $R/2$ ($0.1r_p$ and $1.5r_p$) we also calculate figures for medium masses, i.e. the mass of the perfluoroalkylated PFNS8 nanosphere with 5672 u (u = atomic mass unit), which has been analysed in a interference study [25], and for 10^7 u, representing even bigger nanospheres (for perspective experiments, see [26–29]), because we evaluate matter-wave interferometry with such macromolecules as most appropriate to test our results, see Subsection 4.4. Finally we calculate a density reduction for a macroscopic mass of 1 kg, assuming that in this case a proton is entangled with such a macroscopic object. Whether this latter approach is justified and a realistic application of Eq. (47) remains to be discussed, see the remarks at the end of this section and also Section 5.

Table 1: Approximate calculation of the state reduction (density ρ) for nuclear matter ($m_p = 938.27 MeV/c^2$), using $|\vec{\lambda}|r_p \approx K(n)\,d = 27\pi/2 \times 0.9213$ [2], with the proton charge radius $r_p = 0.84$ fm [1], so $|\vec{\lambda}| \approx 46.5\text{fm}^{-1}$. A is the argument of exp() in Eq. (47) for such values of t that $\cos(\lambda_t ct) = 0$. In the considered limit of $x_5, x_6 \to 0$ λ_t becomes $\lambda_t \approx |\vec{\lambda}|$.

No.	Mass	$R/2$(fm)	$\Delta r/2$(fm)	A	ρ/ρ_0
1	m_p	8.40E−02	0.00	8.23E−03	0.99
2	m_p	8.40E−02	2.80E−02	1.63E−02	0.98
3	m_p	8.40E−02	6.87E−02	1.01E−01	0.90
4	m_p	8.40E−02	8.40E−02	2.05E−01	0.81
5	m_p	1.26E+00	0.00	1.44E−26	1.00
6	m_p	1.26E+00	4.20E−01	2.20E−18	1.00
7	m_p	1.26E+00	1.03E+00	4.82E−06	1.00
8	m_p	1.26E+00	1.26E+00	2.05E−01	0.81
9	5672 u	1.26E+00	0.00	6.36E−22	1.00
10	5672 u	1.26E+00	9.00E−01	4.83E−04	1.00
11	5672 u	1.26E+00	1.00E+00	5.06E−02	0.95
12	5672 u	1.68E+00	0.00	2.08E−30	1.00
13	5672 u	1.68E+00	1.00E+00	1.66E−10	1.00
14	5672 u	1.68E+00	1.40E+00	2.00E−02	0.98
15	10^7 u	1.26E+00	0.00	1.47E−18	1.00
16	10^7 u	1.26E+00	7.50E−01	1.04E−03	1.00
17	10^7 u	1.26E+00	9.00E−01	1.12E+00	0.33
18	10^7 u	1.68E+00	0.00	4.82E−27	1.00
19	10^7 u	1.68E+00	1.00E+00	3.83E−07	1.00
20	10^7 u	1.68E+00	1.30E+00	4.41E−01	0.64
21	1 kg	8.40E−02	0.00	4.91E+24	0.00
22	1 kg	8.40E−02	6.87E−02	6.02E+25	0.00
23	1 kg	1.26E+00	0.00	8.59E+00	0.00
24	1 kg	1.26E+00	1.03E+00	2.88E+21	0.00

4.2 Many body system

Before we analyse the general result of (47), we need to derive an approximate solution for a many body system

21 Not all fermions must fall into the category of the half-integer eigenvalues, but our model allows for this scenario, see Subsection 4.3 in [2]. This would then follow the same rule as other theories of quantum state reduction, see for instance [24], in whose spontaneous collapse model "only non-unitary collapse of fermionic states, and not boson states, is needed to account for the behavior of detectors".

22 Note that $|\vec{\lambda}| \neq \lambda = \lambda(k,l)$ as defined in (8) et sqq.

like the considered nanospheres. In this case the imaginary part on the r.h.s. of Eq. (44), which we name I in the following, must be summed over all particles constituting the mesoscopic system, in our case nucleons, and the position dependent term in function Γ must be understood such that each particle i feels the field ϕ_I of other particles $j, i < j$:[23]

$$I = -2m_f c^2 \sin(\lambda_t ct) \sum_{i<j} e^{-|\vec{\lambda}(\vec{r}_i - \vec{r}_j)|}\, \Psi(\vec{r}_1, \ldots, \vec{r}_N) \quad (49)$$

Because of the quickly decreasing e-function the expression can be well approximated by reducing the sum over j to the nearest neighbour particles of i, which in a simple model means that the sum $S = \sum_{i<j}$ can be limited to all combinations of nucleons in a nucleus and this result be summed over the number of nuclei (i.e. atoms) in the nano object. If we use again $|\vec{\lambda}\vec{r}| \leq |\vec{\lambda}|r$ and further assume that for the distance $|\vec{r}_i - \vec{r}_j|$ between the nucleons a constant mean value r_m can be inserted which should lie around 1.2 fm (see [30] for nuclear radii) to 1.7 fm ($\approx 2r_p$), we get[24]

$$I \approx -2xNm_f c^2 \sin(\lambda_t ct)\, e^{-|\vec{\lambda}|r_m}\, \Psi(\vec{r}_1, \ldots, \vec{r}_N) \quad (50)$$

where N is the number of nucleons in the molecule and $M = Nm_f$ its total mass. The object dependent factor x is the ratio $x = S/N$, $x = 7.84$ for PFNS8 and $x = 10.3$ for a nanosphere consisting of SiO_2 atoms (used as model for the nanosphere of 10^7 u, see [29]). We calculated the density matrix for the nanospheres by inserting this result into Eq. (45) et sqq. with $r, r' = r_m$. According to the range of r_m adopted above, we chose $R/2 = 1.5r_p = 1.26$ fm and $R/2 = 2r_p = 1.68$ fm in Table 1 for these calculations. This approach is certainly still a rough model, but we use it as indication of the order of magnitude of a state reduction in macromolecules by our theory, possibly occurring with superposed nuclear states in such objects with medium mass.

4.3 General result

So, what are the characteristics of our general solution given in reference (47)? The obtained expression of the exponent in (47) generates damping, which is the lower the smaller the expression is. It has a minimum in dependence of $\vec{\Delta}r$ at $\vec{\Delta}r = 0$, i.e. $\vec{r} = \vec{r}\,'$. (This is already true for the general result of (46).) This means that the effect of a density (and thus wave function) reduction is minimal for the diagonal elements, as known from decoherence, but also from stochastic collapse models (see [23, 31, 32]), which were designed to explain the quantum mechanical measurement problem. The difference to these models is that the non-unitary term and thus the exponent does not constantly become zero for the diagonal elements, but varies with a time-dependent function $f = 1 - \cos(\lambda_t ct)$, $0 \leq f \leq 2$, damped by an exponentially decreasing curve $e^{-|\vec{\lambda}\vec{R}|/2}$ ($\rightarrow e^{-|\vec{\lambda}\vec{r}|}$ on the diagonal). Thus, also diagonal elements occasionally "collapse", which is necessary to explain why a reduction of the wave function to a single eigenvalue state happens in a measurement, i.e. in contact with a macroscopic apparatus (see lines 21 and 23 in Table 1). But the fluctuation over time with a frequency of $\lambda_t c \approx 1.4 \cdot 10^{25}\ s^{-1}$ creates a kind of stochastic behaviour which seems to match the known "random" collapse. The dependency of $\vec{R}$ on the other hand shows that diagonal and off-diagonal elements remain more and more stable for bigger radii $\vec{R}$, i.e. for the region in which we consider a linear approximation of Heim's basic equations as possible and thus QM as valid.

We find that a collapse of the wave function occurs a) at very small distances $\leq 0.1\,r_p$, see lines with $R/2 = 0.084$ fm in Table 1; in this case weak damping $\rho/\rho_0 \geq 0.9$ appears on the diagonal and off-diagonal, and is even at $0.05r_p$ not stronger than 0.8, where we are close to end of the scope of our formulae according to the approximation made in (42). These considered distances represent a domain where the effects of the non-linearity of our theory are already noticeable and therefore QM as linear theory should no longer hold. A collapse also occurs b) if $\vec{r} = 0$ or $\vec{r}\,' = 0$, thus $R = \Delta r$, i.e. for an off-diagonal element of the density matrix which points exactly to the centre of the particle (see

23 In analogy to a Coulomb or Yukawa potential applied to a many body system. But this analogy is limited: The imaginary part ϕ_I of field ϕ, thus of ϕ', does not have an imaginary source term in the Euler–Lagrange equation for $L_f + L_{\phi'}$ with derivatives to ϕ'. The source term $\hbar\kappa\overline{\psi}\psi$ is real. This means that Eq. (32) remains homogeneous in ϕ_I, and ϕ_I does not contain a source term with coupling constant $\sim \kappa$. Corresponding to that, the one boson exchange amplitude of perturbative QFT, which would directly describe a fermion-fermion interaction, were quadratic in the coupling constant $-i\hbar\kappa$, but therefore real (and in our case suppressed by the heavy mass of the Higgs propagator). So, the $\vec{r}_i - \vec{r}_j$ dependence in (49) obviously cannot be obtained from QFT. But it can be derived heuristically from Heim's theory: There each particle is a local extremum of field ϕ in the 6-dimensional space [2]. Then each fermion particle interacts with ϕ_I via the coupling of (41), i.e. with all its extrema, representing other particles at positions $\vec{r}_j$. In this picture and the single particle case considered so far, a nucleon (with mass $m_f = m_p$) feels the field $\phi_I \sim e^{-|\vec{\lambda}\vec{r}|}$ at a distance $|\vec{r}|$ from another nucleon. In the subsequent simple approach we treat the nucleons concerning their interaction via ϕ_I as point particles without self-interaction term.

24 Then no i, j dependent term remains under the sum $\sum_{i<j}$ and one obtains $S = 44,496$ for PFNS8 with 156 C and 200 F atoms and $S = 618$ for SiO_2.

lines 4 and 8 in Table 1), and c) if a macroscopic mass is entangled, which is the situation of a measurement with an apparatus, see lines 21–24 in Table 1. In this case in the region of distances $0.2r_\mathrm{p} \le R/2 \le 1.5r_\mathrm{p}$ the damping is off-diagonal orders of magnitude bigger than on the diagonal. Note that despite the macroscopic apparatus mass, the R of the measured particle must lie in this microscopic area, otherwise the factor $\mathrm{e}^{-|\vec{\lambda}\vec{R}|/2}$ nulls the exponent and no collapse occurs. This also holds for the mesoscopic masses which we consider in Table 1 (lines 9–20). Here the situation is more differentiated and to be discussed in the next subsection.

Before, we still note that although the invalidity of linear QM in our theory at the stated very small distances seems undoubted, it remains to be further analysed whether the behaviour of (a) and (b) can be real or is only an "artefact" of our model. But apart from this, in the region $\ge$ the particle radius, our obtained results so far seem to be able to account for the generally observed phenomena, i.e. unitary evolution on- and off-diagonal, but collapse when entangled with a macroscopic mass.

4.4 Experimental test with macromolecules

When it comes to the question how to measure the calculated state reduction effects, it can be concluded from our previous assessment that only experiments with mesoscopic masses seem to offer this opportunity. The other scenarios, i.e. records in Table 1 which provide a density reduction, appear either as inaccessible, like those for the very small radius $R/2 = 0.084$ fm, or imply an instantaneous collapse as it occurs with a macroscopic mass. Hence, a suitable midrange choice of the mass seems to be the only chance to measure a beginning or partial state reduction (i.e. other values of ρ/ρ_0 than 1 or 0).

Macromolecule interferometry[25] offers the possibility to prepare quantum states that are sufficiently well isolated from their environment to avoid decoherence and to show almost perfect coherence [25, 27, 33, 34]. As already stated, we first consider the PFNS8 nanosphere for which (as for other macromolecules) Gerlich et al. [25] have measured quantum interferences with good agreement to the expected amount of visibility.[26] Future experiments with even bigger nanospheres are to be expected [26–29] and may finally give evidence whether and to which quantitative extent quantum interferences are reduced by an "intrinsic" mechanism of state reduction like ours.

According to our calculated results, the PFNS8 and even more a nanosphere of 10^7 u mass, although isolated and thus without environmental decoherence, would lose quantum coherence, i.e. off-diagonal densities be reduced, if Δr grew to more than approx. 70 % of the size of the distance parameter R, see those lines of no. 10–20 in Table 1 with $\Delta r > 0$. However, we must be aware that the results, calculated with use of Eq. (50) and the microscopic distance parameters of nuclear size, describe a scenario with quantum mechanical superpositions of nuclear states which actually should not differ in their spatial location by more than 1.5 fm ($= \Delta r$, i.e. $\Delta r/2 = 0.75$ fm).[27] Thus, taking the results of Table 1 for realistic values of Δr, a state reduction of the considered macromolecules does not happen. This result is obviously supported by the experiments with almost perfect coherence. A mass threshold for a beginning reduction (defined as $\rho/\rho_0 \le 0.99$) as function of Δr and R can be calculated with our resulting formulae and amounts e.g. to $M \approx 6 \cdot 10^{15}$ u $\approx 10^{-2}$ µg for $\Delta r/2 = 0.35$ fm[28] and distance parameter $R/2 = 1.26$ fm. The strong dependency on Δr and R can be illustrated by considering $R/2 = 1.68$ fm instead, but keeping $\Delta r/2 = 0.35$ fm. Then the mass would have to increase to (an unrealistically high) $M \approx 1.8 \cdot 10^{24}$ u $\approx 3\ g$ to reach a reduction.

Finally we recall that according to previous experience, a state reduction, beside through decoherence, only happens in the situation of a measurement, in which macroscopic objects as solids are involved, which can be perfectly described as quantum systems where field quanta as collective modes create robust order over macroscopic distances (compare above). Obviously these QFT based forces inside such solids are much stronger than decohering and "intrinsic" state reduction effects. But when entangled with a microscopic system in a measurement, such a macroscopic object reduces the state of the microscopic system – in accordance with lines 21–24 in Table 1.

25 The author cannot exclude that other approaches like those with Bose–Einstein condensates or superconducting systems (such as SQUIDs), which also constitute many body systems, might be arranged in such a way that the calculated effects become measurable too, but considers this as less promising. For an overview of all relevant experimental areas we refer to [26, 27].

26 $V_\mathrm{observed} = 49 \pm 6\,\%$, $V_\mathrm{expected} = 51\,\%$, classically expected visibility $V_\mathrm{class} < 1\,\%$. So, the experiment would allow a very slight state reduction, but it excludes a bigger reduction effect.

27 We used the deformation parameters $\delta = \Delta r/r_0$ (r_0 is the nuclear radius) of non-spherical nuclei for this estimate [35]. They are ≤ 0.4, thus $\Delta r \le 0.4 \times 3.5$ fm < 1.5 fm for the nuclei of the considered molecules.

28 With deformation parameter $\delta = 0.2$ and nuclear radius 3.5 fm.

5 Discussion

We have shown that core parts of the SM can be derived from Heim's quantum field theory, using only few assumptions. The most important one is the supposed general $U(1)$ symmetry of the additional dimensions x_5, x_6. The resulting local $SU(2) \otimes SU(2) \otimes U(1) \otimes U(1)$ symmetry of Heim's 6-dimensional coordinate space, being isomorphic to the $SU(3)$, is a crucial ingredient from which we deduced the internal degrees of freedom and symmetries of the SM including the thereby generated gauge boson fields. However, in this regard an also more physically founded deduction still should be found. Nonetheless, one should be aware that a R_n space which shall be isomorphic to the gauge symmetry groups of the SM must contain two $U(1)$ symmetric coordinates additionally to the coordinates of the R_4 which build the Lorentz group, locally isomorphic to $SU(2) \otimes SU(2)$. I.e. a 6-dimensional space obviously is suggested if it shall be an "image" of the gauge symmetries.

The poly-metric of Heim's theory provides a further fundamental Eq. (13), from which the Dirac fermion field and equation could be derived by a new approach (compared to [2, 9]). This equation for a single partial metric structure (or superposition of them) provides a linearisation if the composition field, being the sum over all partial structure state functions, is kept in the non-linear term and formally appears as a separate entity. Since further the Higgs field can be identified as a contraction between the composition field and a constant (coordinate independent) vector, the structure of Eq. (13) transforms to a relation between the Dirac and the Higgs field with a Yukawa-like coupling, as in the SM. That this coupling term with the Higgs field, which creates the fermion masses in the SM, results from the non-linear term of Heim's basic equation, which generates the particle masses there (see [2]), seems to be logical. The existence of the Higgs field in the SM could be interpreted as consequence of the non-linearity of the underlying theory.

Interesting is, beside having succeeded in reproducing these structures of the SM, that the stated relation between the partial structure state functions and the composition field as sum of them may shed light on the character of the Higgs field and its relation to the fermion fields. If Heim's theory should be true, then the Higgs field would be an entity resulting from a sum over the partial metric structure state functions, thus fermion fields. Higgs and fermion fields would not be independent of each other, but only derived entities of a fundamental field structure, given by Heim's basic equation and the partial structures of the poly-metric. The parameters of the Yukawa coupling terms were not free, but determined by quantities of the theory. However, measureable quantities like the particle masses cannot be calculated by these new derived relations to the entities of the SM, but only from the solutions of Heim's basic equations [2]. Our derivation from Heim's equations presented here is an approximation which obviously is only good in an "outer" region with $|\vec{\lambda}\vec{r}| > 1$ (compare Eq. (42)) where the non-linear term in the equations is not too strong. This cannot be true in the very microscopic region where the particle masses are determined. Thus, within a model based on Heim's theory, these must be calculated with the full non-linear approach, as done in [2]. Though, that mass calculations of particles composed of quarks in the QCD also exhibit non-linear terms from the self-coupled gluons, shows the same non-linear character.[29]

From our point of view, the obtained results can be interpreted such that Heim's theory with its non-linearity can describe the particles as spacetime "condensations" of very microscopic distances, but it cannot directly depict the microscopic interactions, as the SM does. However, in an asymptotic realm, where the R_4 spacetime is approximately flat, the Heim space shows the same (broken) symmetries as the gauge fields of the SM, so that the existence of these fields can be deduced. In such a picture the gauge interactions are the asymptotic effect of Heim's R_6 in the area of interacting particles.

Our deduction from Heim's basic equations also provided an imaginary term in the coupling between partial structure state function, thus fermion field, and composition field, thus Higgs field (the latter related via (35)). This term turned out to constitute a non-unitary state reduction term in the non-relativistic limit which may address the quantum mechanical measurement problem. Although it is widely accepted that decoherence is a main aspect, as it accounts for the influence of the environment and our ignorance of it, decoherence alone does not yet solve the problem (see [23, 26, 36]). There are various concepts for a solution which we cannot discuss here, but refer to the well-known references of the main approaches (see [31, 32, 37–41]). Our finding in Section 4 obviously falls into the category of modifications to the time-dependent Schrödinger equation,[30] however, it was

29 Interestingly, we found in [2] that the r-dependent course of an interaction potential which can be derived from the *c*- and *d*-hermetry, thus those partial metric structures which should cause the strong interaction in Heim's formalism, seems to have a linear increase in the area between an inner core and the outer asymptotic region, which suggests an analogy to the confinement potential of the QCD.

30 As the theory of Ghirardi, Rimini, and Weber [31] and the subsequent CSL theory [32] or the Schrödinger–Newton equation [39, 41].

not intentionally designed, but derived from a fundamental theory.

Its structure, being non-unitary, having a minimum of reduction at the diagonal matrix elements of the density, but reducing also these diagonal elements (necessary for a collapse to a single eigenstate) with a time-dependent factor, which seems to create a kind of randomness, and being proportional to the involved mass are interesting properties which fit to the observations so far.

Our further analysis, also how to measure the calculated reduction effects, yielded that mesoscopic systems like macromolecules should be best suited to detect such effects. Our current model however, depending sensitively on the (currently) not exactly fixable microscopic distance parameters, cannot yet precisely predict from what mass/size a state reduction will happen. Though, assuming that only small position deviations should be possible in nuclear superpositions, according to our model no state reduction and no loss of interference will be detectable for the up to now tested objects, as the experiments to date confirm. That these interferometry measurements with the so far largest molecule masses [34] still show excellent agreement with quantum theory, implies that further experiments with even much larger objects (isolated from the environment to exclude decoherence) are needed to find out from which mass/size of the objects a state reduction really occurs.

Our model may be enhanced in the following direction to achieve a more detailed description of the interferometry experiments with it: Eq. (45) should be solved including the unitary term for the case of a free particle (analogous to [31], Appendix C). If this solution takes the general form as for other collapse models (see Eq. (1) in [42] = Eq. (3.7) in [31]), then the calculation of the interference pattern itself, developed in [42], can be applied and compared to the experiments, and not only a relative density reduction ρ/ρ_0 by the non-unitary term. Future work should be invested here. In our approach of Subsection 4.2 to apply the obtained imaginary coupling term to a many body system of fermions, it became apparent that, despite the achieved connection to the SM and structures of QFT, it needs to be further explored how a many body system can be described in general by Heim's theory. This is crucial for those areas which go beyond the SM, as treated in Section 4.

Finally we want to point out that the imaginary coupling term has not only its origin in Heim's composition field and thus, according to our derivation, in the Higgs field (its imaginary part), but also relates to the gravitational field, since the composition field merges into the Christoffel symbols of GR in the macroscopic limit. This may draw a connection to those collapse theories which assume gravity as cause of state reduction [39, 41] and may further justify the use of a macroscopic mass in our result to describe the measurement process with an apparatus.[31] But unlike the Schrödinger–Newton equation, which regularly is used in this case as nonrelativistic approximation, our non-unitary solution directly generates a reduction of the density matrix. The characteristics of both approaches is their non-linearity, which in our case yields (amongst others) the imaginary term and thus the quantum state reduction effects described above.

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31 To cite Roger Penrose (p. 855 in [43]): "It is my own standpoint, with regard to quantum state reduction, that it is indeed an objective process, and that it is *always* a gravitational phenomenon. This would be the case even in situations where there has been substantial environmental decoherence …, say in a system (such as a DNA molecule) that is much too small for gravitational OR (*Objective Reduction*) to apply directly to it. In such situations, it would be the total displacement of mass in the environment that results in gravitational OR".

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