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Corrigendum to: The standard model and quantum state reduction from Heim's field theory

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1 Subject of the corrigendum

For the question whether the gauge symmetries of the Standard Model (SM) can be derived from Heim's six-dimensional space as an important aspect of the above-mentioned article, the group-theoretical connection between the Lorentz group and $SU(2) \otimes SU(2)$ as well as that between $SU(3)$ and $SU(2) \otimes SU(2) \otimes U(1) \otimes U(1)$ was investigated. The latter composite group emerges if the two 'trans'-dimensions of the Heim space are added to the four-dimensional spacetime and are supposed to generally follow a $U(1)$ symmetry each.

In the article it was stated that the Lorentz group was locally isomorphic to $SU(2) \otimes SU(2)$ and thus the Heim space to $SU(2) \otimes SU(2) \otimes U(1) \otimes U(1)$ (a) and that, via a Cartan decomposition of the $SU(3)$ to $SU(2) \otimes SU(2) \otimes U(1) \otimes U(1)$ (b), the Heim space was also locally isomorphic to the $SU(3)$. This is incorrect; the group-theoretical relations (a) and (b) each describe a mapping between groups with the same dimension, but the term "isomorphic" (or "isomorphism") is false at this point: the Lorentz group maps to $SU(2) \otimes SU(2)$ [1], [2], but is not compact as the $SU(2)$ is, and a Cartan decomposition of a Lie algebra is not isomorphic to that of the original group.

However, both mappings define a connection (relationship) between the Heim space and the $SU(2) \otimes SU(2) \otimes U(1) \otimes U(1)$ and $SU(3)$ symmetries, which may constitute the first step of a derivation of the SM's gauge symmetries from the Heim space. Why the symmetries manifest themselves in precisely the internal degrees of freedom of the SM, however, remains open for now.

2 Corrected text parts

To implement these clarifications, the subsequent parts of the article are amended as follows (changes in *italic*):

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Abstract: Core parts of the Standard Model are derived from B. Heim's quantum field theory, whose poly-metric describes spacetime and matter in a unified formalism. Its non-linear eigenvalue equation transforms into the Einstein field equation in the macroscopic limit. The six-dimensional Heim space can be determined as locally *mapping* a $SU(2) \otimes SU(2) \otimes U(1) \otimes U(1)$ symmetry, and *thereby also be related* to the $SU(3)$, which allows to connect to the local gauge symmetries and boson fields of the Standard Model. …

3.1 Gauge symmetries

…

Thus, in the asymptotic flat metric the subspace R_4 locally follows the Lorentz group *whose algebra so*$(3,1)$ *is* isomorphic to *the sl*$(2,C)$ (*the su*(2) *is the compact real form of sl*$(2,C)$*). A combination $\vec{A} = \frac{1}{2}(\vec{J} + i\vec{K})$, $\vec{B} = \frac{1}{2}(\vec{J} - i\vec{K})$ of the six generators $\vec{J}$ (rotations) and $\vec{K}$ (boosts) of the Lorentz group leads to two $SU(2)$ groups (with generators $\vec{A}$ and $\vec{B}$) which commute. So the Lorentz group is then essentially* $SU(2) \otimes SU(2)$ (see [1], [2]). *However, it must be noted that this relation is not an isomorphism since $SU(2) \otimes SU(2)$ is compact, but the Lorentz group is not [2].* If now the dimensions x_5 and x_6 in general follow a $U(1)$ symmetry each (i.e. can be rotated in the complex plane, but without change of length), then the six-dimensional space, consisting of the R_4 and the two trans-dimensions, *locally corresponds to a $SU(2) \otimes SU(2) \otimes U(1) \otimes U(1)$ symmetry*. Furthermore, this overall symmetry *is obtained* by a Cartan decomposition of *the* eight-dimensional compact group $SU(3)$ ([3]–[5]). Thus, we have shown the local *mapping* of the six-dimensional Heim space to a duplicated structure of the $SU(2) \otimes U(1)$ and, via these, to the $SU(3)$ if the trans-dimensions x_5, x_6 follow a $U(1)$ symmetry and an asymptotically flat metric is given in the R_4. Then, such a space carries a symmetry which contains a $SU(2) \otimes U(1)$ as a subset, i.e. the gauge symmetry of the electro-weak interaction, and furthermore *relates to* the $SU(3)$, the gauge symmetry of the QCD, *in the manner stated above*. To connect to the SM, we therefore must assume that the Heim space also, as a mapping, manifests in the stated internal degrees of freedom:

$R_6 \Rightarrow$ weak hypercharge, weak isospin, colour

Why it manifests in precisely these internal degrees of freedom remains open.

…

The open question remains whether the $SU(3)$ symmetry then should be broken as well, since it is *connected* via the above stated group theoretical relations *to* the doubled $SU(2) \otimes U(1)$ symmetry.

…

For our current analysis we consider the internal degrees of freedom and symmetries of the SM as being deduced from the *group theoretical connections* above, as well as the symmetry breaking from the arguments given. We consequently apply the related rules including the known covariant derivatives etc. This generates already all gauge boson fields of the SM.

3.4 Yukawa coupling

…

As stated in Subsection 3.1, we currently can only detect the overall *mapping* between the six-dimensional Heim space and the *duplicated* $SU(2) \otimes U(1)$ gauge symmetries, and therefore cannot deduce the concrete implementation with a doublet of left-handed fields/particles at this point, but must adopt it from the phenomenology, as in the SM.

…

5 Discussion

We have shown that core parts of the SM can be derived from Heim's quantum field theory, using only few assumptions. The most important one is the supposed general $U(1)$ symmetry of the additional dimensions x_5, x_6. The resulting *mapping* of Heim's six-dimensional coordinate space *to a* local $SU(2) \otimes SU(2) \otimes U(1) \otimes U(1)$ symmetry, being *related* to the $SU(3)$ *by a Cartan decomposition of the latter*, is a crucial ingredient from which we deduced the internal degrees of freedom and symmetries of the SM including the thereby generated gauge boson fields. *Of course, this deduction is not "necessary", but it is possible and shows a connection between the Heim space and the SM's gauge symmetries.*[1] However, in this regard an also more physically founded deduction still should be found. Nonetheless, one should be aware that a R_n space which shall *"match"* the gauge symmetry groups of the SM must contain two $U(1)$ symmetric coordinates additionally to the coordinates of the R_4 which build the Lorentz *group.* I.e. a six-dimensional space obviously is suggested if it shall be an "image" of the gauge symmetries.…

References

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1 *As mentioned already in [3], in a similar way, Görnitz and Schomäcker deduced the gauge groups $U(1)$, $SU(2)$ and $SU(3)$ as structures of the SM's interactions. But their starting point was an abstract quantum space which carries a $U(1) \otimes SU(2)$ symmetry [4].*